# WOODIN ON THE CONTINUUM PROBLEM: AN OVERVIEW AND SOME OBJECTIONS

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ABSTRACT. I consider W. Hugh Woodin's approach to the Continuum Problem, giving a brief overview of it and discussing a few objections which can be raised. In the last few years, Woodin has developed a new approach to the problem, which has produced such results that although it is not yet solved, one could argue that there are at present clear symptoms that it could have a solution. Woodin's idea is to take an 'incremental' approach: we look for the relevant axioms in turn for the structures $H(\omega)$, $H(\omega_1)$, $H(\omega_2)$ (where as usual $H(\kappa)$ is the set of all sets whose transitive closure has cardinality less than $\kappa$). Any 'complete' axiomatization (in a sense to be defined, taking into account the unavoidable Gödelian incompleteness) for the latter structure yields a solution to the Continuum Problem. The main result obtained by Woodin is that there are such axiomatizations, but any one of them must imply the falsity of the Continuum Hypothesis. More precisely, Woodin's work has led him to the following conjecture: every theory obtained by adding to ZFC an axiom which (a) is compatible with the existence of large cardinals, and (b) makes the properties of sets with hereditary cardinality at most $\aleph_1$ invariant under forcing, implies that the Continuum Hypothesis is false. I summarize Woodin's main results and then discuss some objections which have been raised to his approach by some important working set theorists.

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