A tentative to capture fuzzy logic and approximate reasoning

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Keywords. Approximate reasoning, fuzzy logic, closure operators, effective domains, recursively enumerable fuzzy set, fuzzy control, point-free geometry, similarities.

My researches are devoted to capture the obscure notions of "fuzzy logic" and "approximate reasoning". I try by a limit approximation process "from above" and "from below".

1. From above: the abstract approach.
My starting point are the equations:
logic = effective management of information = a set L of pieces of information + immediate consequence operator which is computable.
In particular, for fuzzy logic we have to define what means information in a vague context and what means effectiveness.
My ideas are exposed in the following:

References

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- L. Biacino, G. Gerla, Fuzzy logic, continuity and effectiveness, Archive for Mathematical Logic, 41, (2002).
At this time I am writing the following manuscript
- G. Gerla, Effective continuous lattices, fuzzy computability and effective immediate consequence operator in approximate reasoning. Unpublished paper.

Obviously, we poach on
Tarski.(Logic, semantics and metamathematics, Oxford (1956)),
D. S. Scott (Continuous lattices, in F. W. Lawvere Editor .... (1972)),
M.B. Smyth, (Effectively given domains, Theoretical Computer Sciences (1977)).
Lawere, (Metric spaces, generalized logic and closed categories, *Rend. Sem. Mat. Fis. Milano* (1973)).
… and others.

2. From below: concrete examples of graded mathematics related with fuzzy logic.

Fuzzy control is the main success in fuzzy set theory. I show that fuzzy control is a chapter of fuzzy programming logic and therefore a chapter of fuzzy logic in narrow sense. Indeed, the fuzzy function arising from a fuzzy system of IF-THEN rules coincides with the last Herbrand model of a suitable fuzzy program.

By interpreting in a multi-valued logic the axioms of reflexivity and transitivity we obtain the definition of a *fuzzy order* as a multivalued interpretation (D, r) where $r : D\times D \rightarrow[0,1]$ is a fuzzy relation satisfying

i) $r(x,x) = 1$ (reflexivity) ;
ii) $r(x,z)\otimes r(z,y)\leq r(x,y)$ (transitivity)

A *fuzzy equivalence* or *similarity* is a multivalued interpretation (D, r) satisfying *i*), *ii*) and
iii) $r(x,y) = r(y,x)$ (symmetry).

These notions are dual of the one of distance and non-symmetric distance. We propose several applications of these notions.

As an example it is possible to unify the *fixed point theory in a metric spaces* with the *fixed point theory in an ordered set*. and it is possible to relate the notion of similarity with the one of *fuzzy group*. Another application is in *point-free geometry*. Indeed, the notion of graded inclusion enable us to resume Whitehead's attempt to base the geometry on the notion of region and inclusion (see *An Inquiry Concerning the Principles of Natural Knowledge* (1919) and *The Concept of Nature* (1920)).

Finally, the multivalued interpretation of similarity suggests the notion of *interval-valued metric space* as a map $\Delta : S\times S \rightarrow I([0,1])$ satisfying suitable axioms. The resulting theory is related in a natural way with interval analysis.

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