# The injectivity of denotational semantics for linear logic proof-nets

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January 14, 2005

**Abstract**

The cut-elimination rules defined on $MELL$ proof-nets give a syntactical equivalence relation, denoted by $\sim_{cut}$, whose canonical representatives are the cut-free proof-nets. On the other hand, the intepretation in a denotational semantics $\mathcal{M}$ gives a semantical equivalence relation, denoted by $\sim_M$.

A model $\mathcal{M}$ is a denotational semantics when $\sim_M$ is invariant by cut-elimination, that is $\sim_{cut} \subseteq \sim_{\mathcal{M}}$. Conversely, $\mathcal{M}$ is *injective* (sometimes called *faithfull*) if $\sim_{cut} \supseteq \sim_{\mathcal{M}}$.

The injectivity is stated as a *completeness* problem ("is $\sim_{cut} = \sim_{\mathcal{M}}$?") in the framework of simple typed $\lambda$-calculus, and it has a positive answer in the theorems of Friedman [1] and Statman [3].

Tortora introduced the problem of injectivity for linear logic proof-nets, obtaining some positive and negative results (see [4], [5] and [2]).

We will analyze the injectivity of coherent and relational semantics, presenting new methods and results. We will state the injectivity in a more dialectic exchange between proof-nets and denotational models, than the severe distinction syntax/semantics, by putting the question in this terms: "what denotational models are able to read from cut-free proof-nets?"

## References

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