Teorema del binomio di Newton

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i} .$$ ← Chiamo questa uguaglianza $P(n)$

Dim. per induzione su n.

<u>Base</u> $n=0$.

$$(x+y)^0 = 1 . \quad \sum_{i=0}^{0} \binom{0}{i} x^i y^{0-i} = \binom{0}{0} x^0 y^{0-0} = 1 .$$

<u>Passo induttivo</u>.

Dimostriamo la formula per n assumendola vera per $n-1$. Ovvero $P(n-1) \Rightarrow P(n)$.

$$(x+y)^n = (x+y)(x+y)^{n-1}$$

$$\underset{\substack{\uparrow \\ \text{uso } P(n-1)}}{=} (x+y) \sum_{i=0}^{n-1} \binom{n-1}{i} x^{n-1-i} y^i$$

(ipotesi induttiva) $$= x\left[\sum_{i=0}^{n-1} \binom{n-1}{i} x^{n-1-i} y^i\right] + y\left[\sum_{i=0}^{n-1} \binom{n-1}{i} x^{n-1-i} y^i\right]$$

(porto x, y dentro le sommatorie) $$= \sum_{i=0}^{n-1} \binom{n-1}{i} x^{n-i} y^i + \sum_{i=0}^{n-1} \binom{n-1}{i} x^{n-1-i} y^{i+1}$$

(Trasformo $\sum_{i=0}^{n-1}$ in $\sum_{j=1}^{n}$) $$= \sum_{j=0}^{n-1} \binom{n-1}{j} x^{n-j} y^j + \sum_{j=1}^{n} \binom{n-1}{j-1} x^{n-j} y^j$$ (pongo $j = i+1$)

$$= \binom{n-1}{0} x^n + \left[\sum_{j=1}^{n-1} \binom{n-1}{j} x^{n-j} y^j\right] + \left[\sum_{j=1}^{n-1} \binom{n-1}{j-1} x^{n-j} y^j\right] + \binom{n-1}{n-1} y^n$$

(osservo che $\binom{n-1}{0} = 1$) $$= x^n + \left[\sum_{j=1}^{n-1} \left(\binom{n-1}{j} + \binom{n-1}{j-1}\right) x^{n-j} y^j\right] + y^n$$

$$= \binom{n}{0} x^n + \sum_{j=1}^{n-1} \binom{n}{j} x^{n-j} y^j + \binom{n}{n} y^n$$

$$= \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j .$$ $\square$

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