Linear Algebra in Curves and Surfaces Modeling

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Geometric modeling is the branch of applied mathematics devoted to methods and algorithms for mathematical description of shapes. Two-dimensional models are of crucial interest in design, technical drawing and computer typography, while three-dimensional models are central to computer-aided-geometric-design (CAGD) and computer-aided-manufacturing (CAM), and widely used in many applied technical fields such as civil and mechanical engineering, architecture, geology, medical image processing, scientific visualization, entertainment. Moreover, since CAGD methods are main ingredients in Isogeometric analysis – an emergent new paradigm for numerical treatment of PDEs which can be seen as a superset of FEMs – it turns out that geometric modeling acquires some relevance also in this area. The main goal of geometric modeling is to create and improve methods, and algorithms for curve and surface representations which is mainly achieved by means of suitable class of functions like splines, or refinable functions to which linear subdivision schemes are associated. For both, the manipulation and the analysis of such a class of functions, several tools of linear algebra play a crucial role like those suited for structured matrices, totally positive matrices, polynomial equations or computation of joint spectral radius. Therefore, aim of this mini-symposium is to gather scientists that, working on different aspects of curves ad surface modeling, face classical and new linear algebra problems and use linear algebra tools to move a step forward in their respective fields.

The 17–th Hilbert's problem and tight wavelet frames

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Tue 16:45, Room Fermi

The 17–th Hilbert's problem was solved in 1927 by Emil Artin. It says that each real, non–negative, multivariate polynomial can be written as a sum of squares of some rational functions. Hilbert in 1888 and Motzkin in 1960 showed that in general one cannot replace rational functions by polynomials in such polynomial representations. In the bivariate case, it is still an open question, the so–called sos problem, if any Laurent polynomial is a sum of squares of some other Laurent polynomials. In the dimention greater or equal to 3, this question has a negative answer as proved by Scheiderer in 1999. In this talk we show how to reduce the problem of constructing of tight wavelet frames, a certain redundant family of functions, to the pure algebraic sos problem. The optimization technique of the semi–definite programing allows us then to check the existence of the corresponding sos representations and to determine them, if they exist. Tight wavelet frames are of special interest as they play an important role in applications such as e.g. signal andimage processing.

Joint work with Joachim Stöckler (TU-Dortmund, Germany)

An algebraic approach to the construction of multichannel wavelet filters

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In previous works [1,2], we proposed *full rank refinable functions* and *multichannel wavelets* as the proper wavelet tools for the analysis of functions which are *vector-valued*. In the orthogonal situation, the matrix filters associated to such functions have to satisfy the so-called *matrix quadrature mirror filter (QMF) equations*, which involve a large number of nonlinear conditions. In this talk we propose an efficient and constructive scheme for finding pairs of matrix solutions to QMF systems. The construction, which extends a procedure given in [3] to the full rank case, mainly makes use of *spectral factorization* techniques and of a matrix completion algorithm based on the resolution of generalized *Bezout identities*. Some examples illustrate the algorithm and the nature of the resulting matrix scaling functions/wavelets.

[1] S. Bacchelli, M. Cotronei, T. Sauer, Wavelets for multichannel signals, Adv. Appl. Math., 29, pp. 581–598, 2002.
[2] C. Conti, M. Cotronei, T. Sauer, Full rank positive matrix symbols: interpolation and orthogonality, BIT, 48, pp. 5–27, 2008.
[3] C. A. Micchelli, T. Sauer, Regularity of multiwavelets, Adv. Comput. Math., 7(4), pp. 455–545, 1997.

Joint work with C. Conti (University of Firenze, Italy)

Approximate Implicitization and Approximate Null Spaces

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The easy conversion of elementary curves and surfaces (lines, circles, ellipses, planes, spheres, cylinders, cones,...) to rational parametric and implicit representations is central in many algorithms used in CAD-systems. For rational Bézier and NURBS-surfaces no such easy conversion exists, a rational parametric surface of bi-degree (n_1, n_2) has in the general case an algebraic degree of $2n_1n_2$, giving the bi-cubic Beziér surface a degree 18 implicit representation. Essential to approximate implicitization is the combination of a rational parametric surface $\mathbf{p}(s,t)$, $(s,t) \in [0,1] \times [0,1]$, with the algebraic surface to be found $q(x,y,z,h) = 0$. The degree m of q, should satisfy $0 < m \le 2n_1n_2$. The combination results in the following factorization, $q(\mathbf{p}(s,t)) = (\mathbf{Db})^T\mathbf{a}(s,t)$, where $\mathbf{b}$ contains the unknown coefficients of $\mathbf{q}$, and $\mathbf{a}(s,t)$ is an array that contains basis function represented in the tensor product Bernstein basis. Similar expressions exist for rational parametric curves and triangular Bézier surfaces. As the Bernstein basis is a partition of unity, and $(s,t) \in [0,1] \times [0,1]$ we have $|q(\mathbf{p}(s,t))| = \left\|(\mathbf{Db})^T\mathbf{a}(s,t)\right\|_2 \le \left\|(\mathbf{Db})^T\right\|_2 \|\mathbf{a}(s,t)\|_2 \le \left\|(\mathbf{Db})^T\right\|_2$. The smallest singular values and their respective coefficient vectors consequently represents alternative implicit approximations to $\mathbf{p}(s,t)$. If $m = 2n_1n_2$ we know that an exact solution exists and that the smallest singular value will be zero. The problem of finding an approximate algebraic representation of $\mathbf{p}(s,t)$ has been refomulated to a problem of finding an approximate null space of the matrix $\mathbf{D}$. One obvious choice is Singular Value Decomposition, however, alternative direct elimination methods also exist.

Joint work with Oliver Barrowclough and Jan B. Thomassen, SINTEF, Oslo, Norway

Structured matrix methods for the construction of interpolatory subdivision masks

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Tue 11:50, Room Fermi

In this talk we discuss the general approach presented in [1] and [2] for the construction of interpolatory subdivision masks by relying upon polynomials and structured matrix computations.

[1]C. Conti, L. Gemignani, L. Romani, From symmetric subdivision masks of Hurwitz type to interpolatory subdivision masks, Linear Algebra Appl., 431, pp. 1971-1987, 2009.
[2] C. Conti, L. Gemignani, L. Romani, From approximating to interpolatory non stationary subdivision schemes with the same reproduction properties, Submitted.

Joint work with C. Conti (University of Florence), L. Romani (University of Milano-Bicocca)

Computing the joint spectral radius in some subdivison schemes.

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Mon 11:00, Room Fermi

In this talk I will consider the analysis of the joint spectral radius of infinite matrix sets arising in the convergence analysis of subdivision schemes [1]. This problem cannot be solved in the general case and presents serious difficulties also from the approximation perspective. Nevertheless it can be handled efficiently when the considered family depends linearly on its parameters. The main tool is the construction of a polyhedral invariant set for the family, which might be obtained in finite time under suitable assumptions. After recalling the main framework [2] and giving some theoretical results I will show some illustrative examples.

[1] N. Guglielmi, C. Manni and D. Vitale, On a class of C^2 Hermite interpolatory subdivision schemes, in preparation.
[2] N. Guglielmi and M. Zennaro, An algorithm for finding extremal polytope norms of matrix families, Linear Algebra and its Applications, vol. 428, pp. 2265–2282, 2008.

Joint work with C. Manni and D. Vitale (University of Roma 2)

Nonnegative Subdivision Revisited

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Tue 11:00, Room Fermi

Recent work by X. L. Zhou, see [3] and the references there, has settled a long-standing question of characterizing convergence of non-negative, univariate subdivision schemes. We relate some of these results to methods used in the analysis of non-homogeneous Markov processes. In particular, the convergence result in [1] (refering to even much older references) is a strong and so far less known basic theorem, from which convergence of nonnegative subdivision can be derived.

We will develope the main ideas and proofs following this approach through properties of stochastic matrices, and of products of families of such matrices. In particular, we will see that we can avoid the notion of the (in general uncomputable) joint spectral radius when dealing with nonnegative subdivision.

[1] J. M. Anthonisse and H. Tijms, Exponential convergence of products of stochastic matrices, *J. Math. Anal.Appl.* **59** (1977), 360–364.
[2] C. A. Micchelli and H. Prautzsch, Uniform refinement of curves, *Lin. Alg. Appl.* **114/115** (1989), 841–870.
[3] X.-L. Zhou, Positivity of refinable functions defined by nonnegative masks, *Appl. Comput. Harmonic Analysis* **27** (2009), 133–156.

Exact calculation of the JSR by depth first search on infinite trees

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Mon 11:50, Room Fermi

We report on our recent progress in computing precisely the joint spectral radius of two matrices. For many subdivision schemes that are relevant in praxis, we are able to specify the exact value of the associated joint spectral radius. Our method is based on a depth first search algorithm on an infinite binary tree whose knots in the k-th level are matrix products of length k. Using a colour coding, this infinite tree has a finite visualisation whose structure can be analysed.

[1] J.Hechler, B.Mößner, U.Reif, C^1-Continuity of the generalized four-point scheme, Linear Algebra and its Applications 430(2009) 3019-3029, Elsevier.

Joint work with Nicole Lehmann (Darmstadt University of Technology) and Ulrich Reif (Darmstadt University of Technology)

Parallel interactive shape modelling and deformation using subdivision surfaces

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Mon 12:15, Room Fermi

Subdivision surfaces provide a compact way to describe a smooth surface using a polygonal model. They are widely used in movie production, commercial modelers and game engines. In these contexts the goal is to enable real-time interactive editing, animation and rendering of smooth surface primitives. To achieve this goal we designed a parallel rendering pipeline which incorporates a special patch-based geometry shader for subdivision surface, integrated with a simple yet effective deformation framework for dynamic exact subdivision surfaces. The field of interactive shape deformation is a very challenging research field, since complex mathematical formulation have to be implemented in a sufficiently efficient and numerical robust manner to allow for interactive applications. Among the surface-based shape deformation techniques we discuss variational optimization and differential coordinates methods which modify differential surface properties instead of spatial coordinates. Linear deformation approaches present Inherent limitations which can be avoided by nonlinear techniques. However, a common drawback of such methods is that the computational effort and numerical robustness are strongly related to the complexity and quality of the surface tesselation.

Recent advances on the applications of totally nonnegative matrices to C.A.G.D.

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It is well known that the bases whose collocation matrices are stochastic and totally nonnegative are the bases in C.A.G.D. with shape preserving properties. We present new applications of totally nonnegative matrices to C.A.G.D. We show that the progressive iteration property related to interpolatory curves has a close relationship with iterative methods applied to totally nonnegative matrices. We also comment some new optimal properties of bases, which are related with extremal properties of their corresponding collocation matrices.

Computing the joint spectral characteristics of large matrices.

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Mon 11:25, Room Fermi

The joint spectral characteristics of matrices such as the joint and lower spectral radii, the p-radius, the Lyapunov exponent, etc., have found many applications, in particular, in the study of refinement equations and subdivision schemes for curves and surfaces design. First we introduce a notion of a general self-similarity equations, whose special case is a refinement equation. Then we show that all joint spectral characteristics of matrices appear naturally as various regularity exponents of solutions of that equation. We consider several approaches for precise and approximate computation of that characteristics for large matrices (as they usually appear in the study of subdivision equations). The methods are based on the analysis of the corresponding extremal norms using tools of convex programming.

Algebraic conditions on non-stationary subdivision symbols for exponential reproduction

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We present an accurate investigation of the algebraic conditions that the symbols of a convergent, binary, linear, non-stationary subdivision scheme should fulfill in order to reproduce spaces of exponential polynomials. A subdivision scheme is said to possess the property of reproducing exponential polynomials if, for any initial data uniformly sampled from some exponential polynomial function, the scheme yields the same function in the limit. The importance of this property is due to the fact that several functions obtained as combinations of exponential polynomials (such as conic sections, spirals or special trigonometric and hyperbolic functions) are of great interest in graphical and engineering applications. Since the space of exponential polynomials trivially includes standard polynomials, the results in this work extend the theory recently developed in [1] to the non-stationary context. As the symbol of the scheme changes from level to level and the parametrization plays a crucial role in this kind of study, the proofs of the non-stationary case are often significantly more difficult and intricate than in the stationary case, and much of the results previously obtained can not be straightforwardly generalized but require a complete reformulation. To illustrate the potentialities of these simple but very general algebraic conditions we will consider affine combinations of known subdivision symbols with the aim of creating new non-stationary subdivision schemes with enhanced reproduction properties.

[1] N. Dyn, K. Hormann, M.A. Sabin, Z. Shen, Polynomial reproduction by symmetric subdivision schemes, J. Approx. Theory, 155, pp. 28-42, 2008.

Joint work with C. Conti (University of Firenze)
