Contributed Talks

A Representation of the Inverse of Tridiagonal Matrices with Nested Functions

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Mon 15:00, Room A

The elements of the inverse of any finite tridiagonal matrix can be expressed in terms of determinants of certain tridiagonal submatrices. This gives simple proofs to known properties, [4], and linear recurrences, [2, 3], of the elements of the inverse of a tridiagonal matrix. A direct representation for the elements of the inverse matrix is also achieved by expressing those determinants in terms of nested functions of the elements of the tridiagonal matrix. This is equivalent to the expressions given in [3]. As an illustration, the resolvent matrix, which arises in spectral theory of finite Jacobi matrices, [1], is detailed.

[1] P. C. Gibson, Inverse spectral theory of finite Jacobi matrices, Trans. Amer. Math. Soc. 354 (2002) 4703-4749.
[2] R.K. Kittappa, A representation of the solution of the n-th order linear difference equation with variable coefficients, Linear Algebra Appl. 193 (1993) 211-222.
[3] R.K. Mallik, The inverse of a tridiagonal matrix, Linear Algebra Appl. 325 (2001) 109-139.
[4] G. Strang, T. Nguyen, The interplay of ranks of submatrices, SIAM Review 46:4 (2004) 637-646.

Joint work with M. Rachidi (Académie de Reims, France)

Which digraphs with ring structure are essentially cyclic?

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Fri 15:25, Room C

The Laplacian matrix of a digraph G with vertex set $V(G) = \{1, \ldots, n\}$ and arc set $E(G)$ is the matrix $L = (\ell_{ij}) \in \mathbb{R}^{n\times n}$ in which, for $j \neq i$, $\ell_{ij} = -1$ whenever $(i, j) \in E(G)$, otherwise $\ell_{ij} = 0$; $\ell_{ii} = -\sum_{j\neq i} \ell_{ij}$, $i, j \in V(G)$.

We say that a digraph is *essentially cyclic* if its Laplacian spectrum is not completely real. The problem of characterizing essentially cyclic digraphs is difficult and yet unsolved. In the present paper, this problem is solved with respect to the class of *digraphs with ring structure*. By such a digraph we mean a digraph that contains a Hamiltonian cycle and whose remaining arcs belong to the inverse Hamiltonian cycle. Two partial results are as follows:

Theorem 1. *Let L_n be the Laplacian matrix of the digraph G_n whose arcs form the Hamiltonian cycle $(1,n)$, $(n,n-1),\ldots,(2,1)$, the path $(1,2)$, $(2,3),\ldots,(i-1,i)$, and the path $(i+1,i+2),\ldots,(n-1,n)$, where $1 \le i < n$. Then:*

1. *The characteristic polynomial of L_n is $\Delta_{L_n}(\lambda) = Z_i(\lambda)Z_{n-i}(\lambda) - (-1)^n$, where $Z_i(x) = (x-2)Z_{i-1}(x) - Z_{i-2}(x)$, $Z_0(x) \equiv 1$, and $Z_1(x) \equiv x-1$.*
2. *If n is even, then G_n is essentially cyclic for all i except for $i = \frac{n}{2}$, in which case the eigenvalues of L_n are $4\cos^2\frac{\pi k}{n}$ and $4\cos^2\frac{\pi k}{n+2}$, $k = 1,\ldots,\frac{n}{2}$.*
3. *If n is odd, then G_n is essentially cyclic for all i except for $i = \frac{n-1}{2}$ and $i = \frac{n+1}{2}$, in which case the eigenvalues of L_n are $4\cos^2\frac{\pi k}{n+1}$, $k = 1,\ldots,n$.*

Theorem 2. *Let G_n be a digraph on $n > 3$ vertices constituted by the cycle $(1,n),(n,n-1),\ldots,(2,1)$ and the opposite cycle $(1,2),(2,3),\ldots,(n-1,n),(n,1)$ in which i $(2 < i < n)$ arcs are missing. Then G_n is essentially cyclic and the Laplacian characteristic polynomial of G_n is $\Delta_{L_n}(\lambda) = \prod_{k=1}^{K} Z_{i_k}(\lambda) - (-1)^n$, where $i_1,\ldots,i_K$ are the path lengths in the decomposition of $(1,n),(n,n-1),\ldots,(2,1)$ into the paths linking the consecutive vertices of indegree 1 in G_n.*

The polynomials $Z_i(x)$ are closely related to the Chebyshev polynomials.

We also consider the problem of essential cyclicity for weighted digraphs.

Joint work with P. Chebotarev (Institute of Control Sciences of RAS)

Investigating the Numerical Range and q-Numerical Range of Non Square Matrices

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Thu 15:00, Room A

Let $\mathcal{M}_{m,n}(\mathbb{C})$ be the algebra of $m\times n$ complex matrices. For $m = n$, $F(A) = \{\langle Ax, x\rangle : x \in \mathbb{C}^n, \|x\|_2 = 1\}$ is the *numerical range* of A [3]. Recently, it has been proposed [2] as numerical range of $A \in \mathcal{M}_{m,n}$ *with respect to* $B \in \mathcal{M}_{m,n}$ the compact and convex set

$$w_{\|\cdot\|}(A,B) = \bigcap_{z_0\in\mathbb{C}} \mathcal{D}(z_0, \|A - z_0B\|). \tag{1}$$

Elaborating the eq.(1), we have noticed that $\bigcup_{\|B\|_F\ge 1} w_{\|\cdot\|_F}(A,B) = \mathcal{D}(0,\|A\|_F)$, thus meaning the independence of $w_{\|\cdot\|_F}(A,B)$ by the matrix B, for $\|B\|_F \ge 1$. Another proposal is the notion of the orthogonal projection onto the lower or higher dimensional subspace and we define *with respect to* an $m\times n$ isometry matrix H $(m \ge n)$: $w_l(A) = F(H^*A)$ *or* $w_h(A) = F(AH^*)$.

In this case, we may have $w(A) = \bigcup_H w_l(A) = \bigcup_H w_h(A)$ and even involving (1), we conclude: $w_l(A) \subseteq w_{\|\cdot\|_2}(A,H) \subseteq w_h(A)$.
Further, we generalize the definition of the numerical range in [1] to the *q-numerical range* of $A \in \mathcal{M}_n$ for $q \in [0,1]$ and we prove for any matrix norm

$$F_q(A) = \bigcap_{z_0\in\mathbb{C}} \mathcal{D}(qz_0, \|A - z_0I_n\|).$$

Hence, we may define the *q-numerical range of* $A \in \mathcal{M}_{m,n}$ *with respect to* $B \in \mathcal{M}_{m,n}$ the set

$$w_{\|\cdot\|}(A,B;q) = \bigcap_{z_0\in\mathbb{C}} \{z \in \mathbb{C} : |z - qz_0| \le \|A - z_0B\|, \; \|B\| \ge q,\; q \in [0,1]\}. \tag{2}$$

Clearly, (2) is a compact and convex set and $w_{\|\cdot\|}(A,B;1) \equiv w_{\|\cdot\|}(A,B)$ in (1).

[1] F.F. Bonsall and J. Duncan, Numerical Ranges II, London Mathematical Society Lecture Notes Series, Cambridge University Press, New York, 1973.
[2] Ch. Chorianopoulos, S. Karanasios and P. Psarrakos, A definition of numerical range of rectangular matrices, Linear Multil. Algebra, **57**, 459-475, 2009.
[3] R.A. Horn and C.R. Johnson, Topics in Matrix Analysis,

Cambridge University Press, Cambridge, 1991.

Joint work with J. Maroulas (National Technical University of Athens)

Structured matrix algorithms for solving the Marchenko integral equations

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The initial-value problem for the focusing nonlinear Schroedinger (NLS) equation

$$\begin{cases} \mathbf{i}\, q_t = q_{xx} + 2q|q|^2, & x \in R,\ t > 0, \\ q(x;0) \quad \text{i.c.}, & x \in R, \end{cases}$$

can be solved by following the various steps of the Inverse Scattering Transform (IST) [1]. Among them, a crucial step consists of the numerical solution of two coupled systems of Marchenko integral equations whose kernels are structured. In fact, their solution uniquely specifies the potential $q(x,t)$ and its energy density at each point $x \in R$ and $t \geq 0$.

We illustrate numerical algorithms for solving the Marchenko systems that take advantage of the Hankel structure of the kernels.

[1] C. van der Mee, Direct and inverse scattering for skew-selfadjoint Hamiltonian systems. In: J.A. Ball, J.W. Helton, M. Klaus, and L. Rodman (eds.), *Current Trends in Operator Theory and its Applications*, Birkhäuser OT **149**, Basel and Boston, 2004, pp. 407–439.

Joint work with S. Seatzu, C. van der Mee, G. Rodriguez (University of Cagliari)

Multivariate and directional majorization on $\mathbf{M}_{n,m}$

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Thu 17:35, Room B

Let $\mathbf{M}_{n,m}$ be the set of all $n \times m$ matrices with entries in $\mathbb{R}$. A square matrix D is called doubly stochastic if it has nonnegative entries and $De = e = D^t e$, where $e = (1,1,...,1)^t$. For $A, B \in \mathbf{M}_{n,m}$, it is said that B is multivariate majorized by A if there exists an $n \times n$ doubly stochastic matrix D such that $B = DA$ and it is said that B is directionally majorized by A if for every $x \in \mathbb{R}^n$ there exists an $n \times n$ doubly stochastic matrix D_x such that $Bx = (D_x)Ax$. It is clear that the multivariate majorization implies the directional majorization but the converse is not true. In this paper we investigate some cases where the multivariate and directional majorization are equivalent on $\mathbf{M}_{n,m}$.

[1] A. Armandnejad, H. Heydari, Linear functions preserving gd-majorization from $\mathbf{M}_{n,m}$ to $\mathbf{M}_{n,k}$. *Bull. Iranian Math. Soc.*, Submmited.
[2] A.W. Marshall, I. Olkin, Inequalities: Theory of Majorization and its Applications, Academic Press , New York, 1979.
[3] F. Martinez Peria, P. Massey and L. Silvestre, Weak matrix majorization, *Linear Algebra Appl.* **403** (2005) 343-368.

Second order pseudospectra of normal matrices

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Thu 11:00, Room A

Let $A \in \mathbb{C}^{n\times n}$ be a normal matrix; a well-known theorem asserts that for all $\varepsilon \geq 0$ the ordinary ε-pseudospectrum of A, $\Lambda_\varepsilon(A)$, is the union of the closed disks of radius ε centered at the eigenvalues of A. We will give a proof of the converse theorem.

Let us define the second order ε-pseudospectrum of any matrix $M \in \mathbb{C}^{n\times n}$ as the set of complex numbers z such that there exists a $\Delta \in \mathbb{C}^{n\times n}$ which satisfies $\|\Delta\| \leq \varepsilon$ and z is a multiple eigenvalue of $M + \Delta$. Let us denote this set by $\Lambda_{\varepsilon,2}(M)$. Here $\|\cdot\|$ stands for the spectral norm.

In this talk we will present a proof of the fact that for any normal matrix A, the set $\Lambda_{\varepsilon,2}(A)$ is a union of closed disks, whose centers and radiuses will be determined in terms of the eigenvalues of A and ε.

[1] M. Karow. *Geometry of spectral value sets.* Ph.D. Thesis, Universität Bremen, 2003.
[2] A.N. Malyshev. A formula for the 2-norm distance from a matrix to the set of matrices with multiple eigenvalues. *Numer. Math.* **83** (3), pp. 443-454, 1999.

Joint work with Juan-Miguel Gracia (The University of the Basque Country, Spain) and Francisco E. Velasco (The University of the Basque Country, Spain)

Spectral regularity of Banach algebras of operators

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Let $\mathcal{B}$ be a Banach algebra with unit element. If D is a bounded Cauchy domain in the complex plane and f is an analytic $\mathcal{B}$-valued function taking invertible values on the boundary ∂D of D, the contour integral

$$\frac{1}{2\pi i}\int_{\partial D} f'(\lambda) f(\lambda)^{-1} d\lambda \qquad (1)$$

is well-defined. By Cauchy's theorem, it is equal to the zero element in $\mathcal{B}$ when f has invertible values on all of D. The Banach algebra $\mathcal{B}$ is said to be *spectrally regular* if the converse of this is true. This means that (1) can only vanish in the trivial case where f takes invertible values on all of D. If $\mathcal{B} = \mathbb{C}$, the integral (1) counts the number of zeros of f inside D; hence $\mathbb{C}$ is spectrally regular. More generally, as a straightforward consequence of a result by A.S. Markus and E.I. Sigal (1970), this also holds for the matrix algebra $\mathbb{C}^{n\times n}$. The Banach algebra $\mathcal{L}(X)$ of all bounded linear operators on an infinite dimensional Banach space X is generally not spectrally regular (example: $X = \ell_2$). In the talk we discuss sufficient conditions for spectral regularity of the Banach subalgebra $\mathcal{L}(X;\mathcal{M})$ of $\mathcal{L}(X)$ consisting of the bounded linear operators on X leaving invariant all members of a given collection $\mathcal{M}$ of closed subspaces of X. New aspects of noncommutative Gelfand theory play a central role.

Joint work with T. Ehrhardt (Santa Cruz, California) and B. Silbermann (Chemnitz, Germany)

On the boundary of the Krein space tracial numerical range

Natalia Bebiano, University of Coimbra, Portugal

Thu 15:25, Room A

Let J be a Hermitian involutive $n \times n$ complex matrix with signature $(r, n-r)$, $0 \leq r \leq n$. We consider $\mathbb{C}^n$ endowed with the indefinite inner product defined by $[x,y] = y^* Jx$, $y, x \in \mathbb{C}$.

For any two $n \times n$ complex matrices C and A, the *J-tracial numerical range* of A (with respect to C), is denoted and defined as:

$$W_C^J(A) = \left\{ \mathrm{tr}(CUAU^{-1}) : \ U \text{ belongs to the } J\text{-unitary group} \right\}$$

This set is connected in the Gaussian plane $\mathbb{C}$, it has a symmetry property, namely $W_C^J(A) = W_A^J(C)$, and several convexity results for this set are known.

In this talk, the boundary generating curve of $W_C^J(A)$ are obtained and the connection between the J-normality of A and the smoothness of $W_C^J(A)$ is deduced.

Joint work with H. Nakazato (Hirosaki University), Ana Nata (Polytechnic Institute of Tomar, Portugal), J. P. da Providência (University of Coimbra, Portugal)

Computing the block factorization of complex Hankel matrices: application to the Euclidean algorithm

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Fri 15:25, Room B

In this work, we present an algorithm for finding an approximate block diagonalization of complex Hankel matrices via an inversion techniques of an upper triangular Toeplitz matrix, specifically, by simple forward substitution. Our method is based on the results of [1] for computing an approximate block diagonalization of real Hankel matrices. We also consider an approximate block diagonalization of complex Hankel matrices via Schur complementation. An application of our algorithm by calculating the "approximate" polynomial quotient and remainder appearing in the Euclidean algorithm is also given. We have implemented our algorithms in Matlab. Numerical examples are included. They show the effectiveness of our strategy.

[1] S. Belhaj, A fast method to block-diagonalize a Hankel matrix, Numer. Algor., 47, pp. 15–34, 2008.

Matrix Polynomials in the Max Algebra; Eigenvalues, Eigenvectors and Inequalities

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The max algebra consists of the set of nonnegative real numbers together with two binary operations: maximization denoted by $\oplus$ and multiplication denoted by $\otimes$. Matrix operations over the max algebra are defined in the natural manner. We consider matrix polynomials of the form

$$P(\lambda) = A_0 \oplus \lambda A_1 \oplus \cdots \oplus \lambda^{m-1} A_{m-1}$$

where $A_0, A_1, \ldots, A_{m-1} \in \mathbb{R}^{nxn}$ are nonnegative matrices. Specifically, in the sprit of [1], we first present a version of the Perron-Frobenius Theorem [2] for polynomials of this type. Applications of this result to the convergence properties of multistep difference equations over the max algebra are also described. Finally, we discuss the relation between $\mu(P(\lambda))$, the largest max eigenvalue of $P(\lambda)$, and the maximal cycle geometric mean, $\mu(P(1))$, of the nonnegative matrix $P(1)$. Several inequalities relating $\mu(P(\lambda))$ and $\mu(P(1))$, echoing similar results for the conventional algebra, are described.

[1] P. J. Psarrakos and M. J. Tsatsomeros, A primer of Perron- Frobenius theory for matrix polynomials, Linear Algebra Appl. 393 (2004) 333-351.
[2] R.B. Bapat, A max version of the PerronFrobenius theorem, Linear Algebra Appl. 275-276 (1998) 3-18.

Joint work with Oliver Mason (Hamilton Institute, National University of Ireland, Maynooth)

The matrix equation $XA - AX = f(X)$

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Thu 15:25, Room C

Let f be an analytic function defined on a complex domain Ω and $A \in \mathcal{M}_n(\mathbb{C})$. We assume that there exists unique α satisfying $f(\alpha) = 0$. When $f'(\alpha) = 0$ and A is nonderogatory, we solve completely the equation $XA - AX = f(X)$. This generalizes Burde's result. When $f'(\alpha) \neq 0$, we give a method to solve completely the equation $XA - AX = f(X)$: we reduce the problem to solve a sequence of Sylvester equations. Solutions of the equation $f(XA - AX) = X$ are also given in particular cases.

The importance of a dummy paper

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Wed 11:50, Room A

Link analysis has been proposed recently as a tool for ranking scientific publications. For example, in [1,2] a collection of papers is modeled as the states of a Markov chain, with a transition probability associated with every citation. To enforce regularity, the chain is modified by adding a state associated to a *dummy paper*, which cites and is cited by all the papers in the collection. Paper ranking is obtained by computing the invariant probability vector of the modified chain. Not very surprisingly, in this model the dummy paper receives the highest score.

In similar contexts, as the Google search engine or the EigenFactor bibliometric index, regularity of the Markov chain is obtained by allowing random jumps from every state to every other state, performed with a prescribed probability usually tuned by means of a parameter $0 \leq \alpha < 1$.

In this talk we show that the two approaches give rise to two out of a wider family of models, depending on n parameters $0 \leq \alpha_i < 1$, where $i = 1, \ldots, n$ and n is the number of states. The parameter α_i tunes the probability of the random jump from state i or, equivalently, the probability of the transition from the i-th paper to the dummy paper. These parameters can be used to introduce time dependent features in the models e.g., by lowering parameter values of older states. Within this family of models, we study the problem of node updating, which for a generic Markov chain is quite difficult. We show that a certain subfamily, which includes the dummy paper model, has desirable properties from this point of view, generalizing a result presented in [1].

[1] D. A. Bini, G. Del Corso, F. Romani. A combined approach for evaluating papers authors and scientific journals, *Technical Report TR-08-10, Dipartmento di Informatica, University of Pisa*, 2008.
[2] D. A. Bini, G. Del Corso, F. Romani. Evaluating scientific products by means of citation-based models: a first analysis and validation *ETNA* 33 (2008-2009), 1–16.

Joint work with D. Fasino (University of Udine)

Algebraic reflexivity for semigroups of operators

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Tue 11:00, Room B

Let V be a vector space over a field $\mathbb{F}$. For a non-empty set $\mathcal{T}$ of linear transformations on V, let $\mathrm{Lst}\,\mathcal{T}$ be the family of all $\mathcal{T}$-invariant subsets of V. For a non-empty family $\mathfrak{M}$ of subsets of V, let $\mathrm{Sgr}\,\mathfrak{M}$ be the set of all linear transformations T on V satisfying $\mathfrak{M} \subset \mathrm{Lst}\,T$. Then $\mathrm{Lst}\,\mathcal{T}$ is a lattice with respect to the taking unions and intersections. $\mathrm{Sgr}\,\mathfrak{M}$ is a multiplicative semigroup of linear transformations. It is easily seen that $\mathcal{T} \subseteq \mathrm{Sgr}\,\mathrm{Lst}\,\mathcal{T}$. A multiplicative semigroup $\mathcal{S}$ of linear transformations is said to be algebraicaly reflexive if $\mathrm{Sgr}\,\mathrm{Lst}\,\mathcal{S} = \mathcal{S}$.

We study algebraic reflexivity of multiplicative semigroups of linear transformations and give some examples of algebraic reflexive semigroups. At the end we characterize those bounded linear operators on a complex Banach space that are determined by the lattice of invariant subsets.

[1] J. Bračič, Algebraic reflexivity for semigroup of operators, Electron. J. Linear Algebra, 18, pp. 745-760, 2009.

Lorentzian Distance Matrices

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Tue 11:50, Room A

We consider distance matrices in the Lorentzian n-space, $\mathbb{R}^{1,n-1}$. A matrix $D = [d_{ij}]_{i,j=1,\ldots,m}$ is said to be a Lorentzian distance matrix if there exists a set of points of $\mathbb{R}^{1,n-1}$, $\mathcal{X} = \{x_1, x_2, \ldots, x_m\}$, such that $d_{ij} = ||x_i - x_j||_\circ^2$, where $||.||_\circ$ denotes the Lorentzian norm.
In this study we present an alternative proof for a classical characterization, due to [1], of this type of matrices. Other characterizations are also taken into consideration. It is known that every Euclidian distance matrix is an elliptic matrix, we prove that every elliptic matrix is a Lorentzian distance matrix. With this framework, we investigate how to distinguish the elliptic matrices that are strictly Lorentzian (*i.e.*, non Euclidian).

[1] I. J. Shoenberg. Remarks to Maurice Frechet's Article "Sur La Definition Axiomatique D'Une Classe D'Espace Distances Vectoriellement Applicable Sur L'Espace De Hilbert". *The Annals of Mathematics, 2nd Ser.*, Vol. 36, No. 3. (Jul., 1935), pp. 724-732.

Joint work with A. Breda (University of Aveiro)

On the gaps in the set of exponents of primitive boolean circulant matrices

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It is well-known that the maximum exponent that an n-by-n boolean primitive circulant matrix can attain is $n-1$. We consider the problem of describing the possible exponents attained by these kind of matrices. This problem is equivalent to the following two problems: 1) finding the set of exponents attained by primitive Cayley digraphs on a cyclic group ; 2) determining the set of orders of bases for $\mathbb{Z}_n$. We present a conjecture for the possible such exponents and prove this conjecture in several cases. We also find the maximum exponent that n-by-n boolean primitive circulant matrices with constant number of nonzero entries in its generating vector can attain and give matrices attaining such exponents.

Joint work with S. Furtado (Faculdade de Economia do Porto, Portugal)

Naturally graded n-dimensional Leibniz algebras of nilindex $n-3$.

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Fri 17:10, Room Galilei

Leibniz algebras present a "non commutative" analogue of Lie algebras and they were introduced by J.-L. Loday, [5], as algebras which satisfy the following Leibniz identity: $[x,[y,z]] = [[x,y],z] - [[x,z],y]$.

It should be noted that Lie algebras are particular cases of Leibniz algebras. For a given Leibniz algebra L we consider lower central series: $L^1 = L$ and $L^{k+1} = [L^k, L^1], \quad k \geq 1$.

A Leibniz algebra L is called nilpotent if there exists $s \in \mathbb{N}$ such that $L^s = \{0\}$. The minimum number satisfying this property is called the nilindex of L. For an n-dimensional Leibniz algebra, we have the natural filtration:

$$L \supseteq L^2 \supseteq \cdots \supseteq L^{n-3} \supseteq L^{n-2} \supseteq L^{n-1} \supseteq L^n \supseteq L^{n+1} = \{0\}.$$

Then the description of n-dimensional algebras L with the following conditions: $L^{n-i} \neq \{0\}, L^{n-i+1} = \{0\}, 0 \leq i \leq n-1$ for any value of i gives pairwise non isomorphic classes of algebras, more precisely, for different i the defined classes of algebras are disjoint. Evidently, the nilindex of an n-dimensional algebra does not exceed $n+1$. A Leibniz algebra is called zero-filiform, filiform and quasi-filiform, if its nilindex is equal to $n+1$, n and $n-1$, respectively. The classification of naturally graded algebras is obtained already. In other words, n-dimensional naturally graded Leibniz algebras with length of the natural filtration equal to $n+1$, n and $n-1$ are known [1], [2], [3] and [4]. The descriptions of some subclasses of naturally graded Leibniz algebras with length of the filtration $n-2$ were obtained. The main result of this work is to complete the classification of complex n-dimensional naturally graded Leibniz algebras with length of the filtration equal to $n-2$.

[1] Sh.A. Ayupov, B.A. Omirov, On some classes of nilpotent Leibniz algebras, (Russian) Sibirsk. Mat. Zh., 42 (1), pp. 18-29, 2001; translation in Siberian Math. J., 42 (1), pp. 15-24, 2001.
[2] L.M. Camacho, J.R. Gómez, A.J. González, B.A. Omirov, Naturally graded quasi-filiform Leibniz algebras, Journal of Symbolic Computation, 44, pp. 527–539, 2009.
[3] L.M. Camacho, J.R. Gómez, A.J. González, B. A. Omirov, Naturally graded 2-filiform Leibniz algebras, Communications in Algebra, To appear.
[4] J.R. Gómez, A. Jiménez-Merchán, Naturally graded quasi-filiform Lie algebras, J. Algebra, 256(1), pp. 211-228, 2002.
[5] J.L. Loday, Une version non commutative des algébres de Lie: les algébres de Leibniz, Ens. Math., 39, pp. 269-293, 1993.

Joint work with L.M. Camacho (Universidad Sevilla), J.R. Gómez (Universidad Sevilla), Sh.B. Redjepov (Institute of Mathematics and Information Technologues, Uzbekistan Academy of Science)

Block-diagonal stability for switched systems

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A switched linear system is a family of time invariant linear systems, called the system bank, together with a switching law that determines how the time invariant systems commute among themselves. We consider switched systems with a finite system bank $\{\Sigma_p = (A_p, B_p, C_p, D_p) : p \in \mathcal{P}\}$, where $\mathcal{P}$ a finite index set. It is a well-known fact that the existence of a positive definite matrix P such that $A_p^T P + PA_p < 0$, for all $p \in \mathcal{P}$, implies the stability of the overall switched system, under arbitrary switching. In this case, the time invariant system Σ_p are said to have a common quadratic Lyapunov function.

In this work, we assume that the system matrices A_p have a pre-specified block structure and we investigate the existence of a common quadratic Lyapunov function with block-diagonal structure.

[1] Isabel Brás, Ana Carapito, Paula Rocha, Block-diagonal stability for switched systems, *In preparation.*

Joint work with Isabel Brás (Universidade de Aveiro) and Paula Rocha (Universidade do Porto)

The set of feedback assignable polynomials to a non-controllable single-input linear system

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A canonical form for generic single input linear systems over a Bézout domain R (including non-reachable/non-controllable cases) is given. This canonical form can be used to compute effectively the set of assignable polynomials of a given linear system and some feedback invariants.

We also generalize a classical result in control theory by proving that given a Bézout domain and a single input linear system $\Sigma = (A, \underline{b}) \in R^{n\times n} \times R^{n\times 1}$, the smallest principal ideal $(q) \subseteq R[z]$ of $R[z]$ containing $\mathcal{U}_n(z\mathbf{1} - A, \underline{b})$ is a feedback invariant and divides all feedback assignable polynomials to Σ.

(k, τ)-regular sets of circulant graphs

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Fri 15:00, Room C

Given a graph $G = (V(G), E(G))$, a subset of vertices $\emptyset \neq S \subseteq V(G)$ is a (k, τ)-regular set if S induces a k-regular subgraph in G and every vertex in $V(G)\backslash S$ has exactly τ neighbors in S. In this presentation we introduce some results on the characterization of (k, τ)-regular sets for circulant graphs with symbol that fulfills some requirements and we prove the existence of (k, τ)-regular sets for certain values on the order of G, namely, $|V(G)|$ even and $|V(G)|$ multiple of 3. According to [1], a subset $\emptyset \neq S \subseteq V(G)$ of a regular graph is a (k, τ)-regular set if and only if $k - \tau$ is an eigenvalue of G. Since circulant graphs are regular graphs, from the above results we obtain a combinatorial characterization of the spectrum of circulant graphs.

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Joint work with P. Rama (University of Aveiro)

Low-Rank Approximation of Graph Similarity Matrices

Thomas P. Cason, Université catholique de Louvain, Belgium
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Mon 11:50, Room A

Graphs are a powerful tool for many practical problems such as pattern recognition, shape analysis, image processing and data mining. Measures of graph similarity have a broad array of applications, including comparing chemical structures, navigating complex networks like the World Wide Web, and analyzing different kinds of biological data [1].

Blondel *et al.* introduced the notion of similarity between nodes of two graphs in [2]. They defined a similarity measure as a fixed point of the even iterates of the following recurrence

$$S_0 = \mathbf{1}_{m,n} , \qquad S_{k+1} = \mathcal{M}(S_k)/\left\|\mathcal{M}(S_k)\right\|,$$

where $\mathcal{M}(S) := ASB^T + A^T SB$ and A and B are graph adjacency matrices. One can prove that the similarity matrix is solution of $\max_{\langle S,S\rangle=1} \Phi(S) = \mathrm{tr}\left(S^T\mathcal{M}^2(S)\right)$. When S is large, the iteration becomes computationally expensive. Hence one can think to modify the problem in order to find an approximation of S at lower cost. In this work, we consider the approximation of the similarity matrix S in $\mathcal{S}$, the set of matrices of norm 1 and rank at most k.

We propose the following algorithm to find stationary points of Φ

$$S_+ := \arg\max_{\tilde{S}\in\mathcal{S}} \mathrm{tr}\left(\tilde{S}^T\mathcal{M}^2(S)\right) \tag{1}$$

The maximum is achieved when S is aligned with the dominant space of $\mathcal{M}^2(S)$. One iteration of (1) costs

$$6\ (m^2+n^2)\ k + 17\ (m+n)\ k^2 + O(k^3)$$

whereas one full rank iteration costs $4(m^2n + n^2m)$.

We characterize the fixed points of (1) and prove that all accumulation points are stationary points of $\Phi(S)$. Preliminary results were presented in [3].

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[3] T. Cason, P.-A. Absil, and P. Van Dooren. Iterative methods for low rank approximation of graph similarity matrices. Presented at 7th MLG, 2009.

Joint work with P.-A. Absil and P. Van Dooren (UC Louvain, Belgium)

Perturbation analysis of Markov chains: a matrix calculus approach

Hal Caswell, Woods Hole Oceanographic Institution, USA
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Wed 11:50, Room B

Whenever Markov chains are used as models of real-world phenomena, perturbation analysis, quantifying the sensitivity of conclusions to changes in parameters, is an important problem. The Magnus-Neudecker formalism for matrix calculus provides easily computable solutions for many such problems. Here, I will summarize some recent results on the perturbation analysis of absorbing and ergodic finite-state Markov chains. In absorbing chains, interest focuses on questions related to the time to absorbtion, and a key to these results is the fundamental matrix $\mathbf{N} = (\mathbf{I} - \mathbf{U})^{-1}$, where $\mathbf{U}$ is the matrix of transition probabilities among the transient states. Suppose that $\mathbf{U}$ is a function of a parameter vector θ. Then it can be shown that

$$\frac{d\mathrm{vec}\mathbf{N}}{d\theta^{\mathsf{T}}} = \left(\mathbf{N}^{\mathsf{T}} \otimes \mathbf{N}\right) \frac{d\mathrm{vec}\mathbf{U}}{d\theta^{\mathsf{T}}} \tag{1}$$

where the vec operator stacks columns of a matrix one above the next, and $\otimes$ denotes the Kronecker product. Extensions of this result will be shown for the sensitivity and elasticity of the moments of the time to absorbtion, for discrete- and continuous-time absorbing chains. When applied to ergodic chains, the approach yields the sensitivity of the stationary distribution $\hat{\mathbf{p}}$. Let $\mathbf{P}$ be the transition matrix, assumed to be a function of a parameter vector θ; then

$$\frac{d\hat{\mathbf{p}}}{d\theta^{\mathsf{T}}} = \left(\mathbf{I} - \mathbf{P} + \hat{\mathbf{p}}\mathbf{e}^{\mathsf{T}}\mathbf{P}\right)^{-1} \left(\hat{\mathbf{p}}^{\mathsf{T}} \otimes \mathbf{I} - \hat{\mathbf{p}}^{\mathsf{T}} \otimes \hat{\mathbf{p}}\mathbf{e}^{\mathsf{T}}\right) \frac{d\mathrm{vec}\mathbf{P}}{d\theta^{\mathsf{T}}} \tag{2}$$

where $\mathbf{e}$ is a vector of ones. I will illustrate the results with some ecological and demographic applications.

Combinatorial Identities from LU Decomposition of Matrices

Marc Chamberland, Grinnell College, USA
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Mon 11:25, Room A

The LU decomposition is a standard tool used in numerical linear algebra. This talk shows how this tool may be used to obtain combinatorial identities, some of which are new. As a simple example, choose the (i, j) entry of a 6×6 matrix to be F_{i+j-1}^2, where F_k is the k^{th} Fibonacci number. An LU decomposition produces

$$\begin{bmatrix} 1 & 1 & 4 & 9 & 25 & 64 \\ 1 & 4 & 9 & 25 & 64 & 169 \\ 4 & 9 & 25 & 64 & 169 & 441 \\ 9 & 25 & 64 & 169 & 441 & 1156 \\ 25 & 64 & 169 & 441 & 1156 & 3025 \\ 64 & 169 & 441 & 1156 & 3025 & 7921 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 4 & 5/3 & 1 & 0 & 0 & 0 \\ 9 & 16/3 & 2 & 1 & 0 & 0 \\ 25 & 13 & 6 & 0 & 1 & 0 \\ 64 & 35 & 15 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 & 9 & 25 & 64 \\ 0 & 3 & 5 & 16 & 39 & 105 \\ 0 & 0 & 2/3 & 4/3 & 4 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Recognizing the terms in the factored matrices yields the identity

$$F_{i+j-1}^2 = F_i^2 F_j^2 + \frac{1}{3} F_{i-1} F_{i+2} F_{j-1} F_{j+2} + \frac{2}{3} F_{i-2} F_{i-1} F_{j-2} F_{j-1}$$

Many diverse identities will be given by performing an LU decomposition on matrices whose terms involve binomial coefficients, number theoretic functions, orthogonal polynomials, q-series, and multiple derivatives of functions.

Graph Laplacians and Logarithmic Forest Distances

P. Chebotarev, Institute of Control Sciences of the RAS, Russia
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Fri 17:35, Room C

A new parametric family of distances for graph vertices is proposed. At the extreme values of the parameter, the family generates the shortest-path distance and the resistance distance (coinciding with the commute time distance). A distinctive feature of the family members is that they are graph-geodetic: $d(i, j) + d(j, k) = d(i, k)$ if and only if every path from i to k passes through j. The family is constructed as follows:

$$Q_\alpha = (I + \alpha L)^{-1},$$

where $\alpha \in \mathbb{R}_+$ is a parameter and L is the Laplacian matrix of the graph,

$$H_\alpha = \gamma\,(\alpha - 1)\,\overrightarrow{\log_\alpha Q_\alpha},$$

where $\alpha \neq 1$, $\gamma \in \mathbb{R}_+$, and the logarithm $\overrightarrow{\log_\alpha Q_\alpha}$ is taken entrywise, and finally,

$$D_\alpha = \tfrac{1}{2}(h_\alpha \mathbf{1}^{\mathrm{T}} + \mathbf{1} h_\alpha^{\mathrm{T}}) - H_\alpha,$$

where h_α is the column of the diagonal entries of H_α and $\mathbf{1} = (1, \ldots, 1)^{\mathrm{T}}$, provides the matrix of distances. The proofs of the properties of the family [1] involve the matrix forest theorem [2] and the graph bottleneck inequality [3]. On the possible applications, see [4]. A sensible choice of the scaling parameter γ is $\gamma = \ln(e + \alpha^{\frac{2}{n}})$. The distances are called the *logarithmic forest distances.*

[1] P. Chebotarev, A family of graph distances generalizing both the shortest-path and the resistance distances, arXiv preprint math.CO/0810.2717. http://arXiv.org/abs/0810.2717
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[4] L. Yen, M. Saerens, A. Mantrach, M. Shimbo, A family of dissimilarity measures between nodes generalizing both the shortest-path and the commute-time distances, 14th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, pp. 785–793, 2008.

Eigenpairs of Adjacency Matrices of Balanced Signed Graphs

Mei-Qin Chen, Department of Mathematics and Computer Science, The Citadel
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Thu 16:45, Room C

In this paper, we present results on eigenvalues λ and their associated eigenvectors x of an adjacency matrix A of a balanced signed graph. A graph $G = (V, E)$ consists of a set V of vertices and a set E of edges between two adjoined vertices. A signed graph is a graph for which each edge is labeled with either $+$ or $-$. A signed graph is said to be balanced if there are an even number of negative signs in each cycle (a simple closed path).

Signed graphs were first introduced and studied by F. Harary to handle a problem in social psychology. It was shown by Harary in 1953 that a signed graph is balanced if and only if its vertex set V can be divided into two sets (either of which may be empty), X and Y, so that each edge between the sets is negative and each within a set is positive. Based on this fundamental theorem for balanced signed graphs, vertices of a balanced signed graph can be labeled in a way so that its adjacency matrix is well structured. Using this special structure, we find exactly all eigenvalues and their associated eigenvectors of the adjacency matrix A of a given balanced signed graph. We will present eigenpairs (λ, x) of adjacency matrices of three types of balanced signed graphs: (1) graphs that are complete; (2) graphs with t vertices in X or in Y that are not connected; and (3) graphs that are bipartite.

Joint work with Spencer P. Hurd (The Citadel)

On classical adjoint-commuting mappings between matrix algebras

Wai-Leong Chooi, University of Malaya, Malaysia.
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Fri 17:35, Room Galilei

Let $\mathbb{F}$ be a field and let m and n be integers with $m, n > 2$. Let $\mathcal{M}_n$ denote the algebra of $n \times n$ matrices over $\mathbb{F}$. In this note, we characterize mappings $\psi : \mathcal{M}_n \to \mathcal{M}_m$ that satisfy one of the following conditions:

1. $|\mathbb{F}| = 2$ or $|\mathbb{F}| > n$, and $\psi(\mathrm{adj}\,(\mathrm{A} + \alpha\mathrm{B})) = \mathrm{adj}\,(\psi(\mathrm{A}) + \alpha\psi(\mathrm{B}))$ for all $A, B \in \mathcal{M}_n$ and $\alpha \in \mathbb{F}$ with $\psi(I_n) \neq 0$.
2. ψ is surjective and $\psi(\mathrm{adj}\,(\mathrm{A} - \mathrm{B})) = \mathrm{adj}\,(\psi(\mathrm{A}) - \psi(\mathrm{B}))$ for all $A, B \in \mathcal{M}_n$.

Here, adj A denotes the classical adjoint of the matrix A, and I_n is the identity matrix of order n. We give examples showing the indispensability of the assumption $\psi(I_n) \neq 0$ in our results.

Joint work with Wei-Shean Ng (Universiti Tunku Addul Rahman, Malaysia.)

A numerical range for rectangular matrices and matrix polynomials

Ch. Chorianopoulos, National Technical University of Athens, Greece
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Tue 15:00, Room B

The numerical range of an operator can be written as an (infinite) intersection of closed circular discs. This interesting property was observed by Bonsall and Duncan (1973), and leads (in a natural way) to a definition of numerical range of rectangular complex matrices. The new range is always compact and convex, and satisfies basic properties of the standard numerical range. The proposed definition is also extended to the case of matrix polynomials.

Joint work with P. Psarrakos (National Technical University of Athens)

Solution of Non-Symmetric Algebraic Riccati Equations from Transport Theory

Eric King-wah Chu, Monash University, Melbourne, Australia
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Tue 15:50, Room Fermi

Transport theory [1,3] provides a rich source of mathematical problems. For example, from (i) a differential-integral equation in a two-dimensional model, or (ii) a differential equation in a one-dimensional multi-state model, we shall derive and study the non-symmetric algebraic Riccati equation

$$B^- - XF^- - F^+X + XB^+X = 0,$$

where (i) $F^{\pm} \equiv I - \hat{s}PD^{\pm}$, $B^- \equiv (\hat{b}I + \hat{s}P)D^-$ and $B^+ \equiv \hat{b}I + \hat{s}PD^+$ with positive diagonal matrices $D^{\pm}$, a low-ranked P and positive parameters $\hat{b}$ and $\hat{s}$; or (ii) $F^{\pm} \equiv (I - F)D^{\pm}$ and $B^- \equiv BD^-$ with possibly low-ranked matrices F and B. These are generalizations of the one studied by Juang in [2].

We prove the existence of the minimal solution X^* under physically reasonable assumptions, and study its numerical computation by fixed point and Newton iterations. We shall also study several special cases. For example, when (i) $\hat{b} = 0$ and P is low-ranked, then $X^* = \hat{s}UV^\top$ is low-ranked; or (ii) when B and F are low-ranked, then $X^* = T \circ (UV^\top)$ with the low-ranked $UV^\top$. The solution can then be computed using more efficient iterative processes. Numerical examples will be given to illustrate our theoretical results.

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[3] G. M. Wing, *An Introduction to Transport Theory*, Wiley, New York , 1962.

Joint work with J. Juang (National Chiao Tung University), T. Li (Southeast University), and W.-W. Lin (National Chiao Tung University)

On normal Hankel matrices

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Mon 15:25, Room A

The normal Hankel problem is the one of characterizing the matrices that are normal and Hankel at the same time. This problem turned out to be much harder than the normal Toeplitz problem, which the authors solved in early 1990's.

In this talk, we give a sketch of the complete solution of the normal Hankel problem. We present a general approach that allows us to obtain all the classes of normal Hankel matrices as special cases of a unified scheme.

Joint work with Kh. D. Ikramov (Faculty of Computational Mathematics and Cybernetics, Moscow State University, Leninskie gory, Moscow, 119992 Russia)

A Lower Bound for the Distance from a Controllable Switched Linear System to an Uncontrollable One

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Tue 15:25, Room A

We consider the set of controllable switched linear systems (SLS). Since the parameters of a given mathematical model are usually determined only approximately, an uncontrollable system may appear as a controllable one. In other words, in general an uncontrollable system becomes controllable when perturbing. Nevertheless, the converse may also occur if perturbations are big enough. In this work we obtain a lower bound for the distance from a controllable SLS to the nearest SLS which is uncontrollable, thus determining a safety neighbourhood for any controllable SLS.

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[3] J. Clotet, Mª Isabel García-Planas, M. D. Magret, Estimating distances from quadruples satisfying stability properties to quadruples not satisfying them, Linear Algebra

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[5] R. Eising, Between controllable and uncontrollable, Syst. Control Letters 4, no. 5, pp. 263-264, 1984.
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[7] Z. Sun, S. S. Ge, Switched Linear Systems, London, England. Springer, 2005.

Joint work with J. Ferrer, M. D. Magret (Universitat Politècnica de Catalunya)

What's New? Matrix Methods for Extracting Update Summaries

John M. Conroy, IDA Center for Computing Sciences, Bowie, MD, USA
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Wed 11:00, Room A

In this talk we will describe the use of linear algebra to develop algorithms to extract information from text documents. The problem is two-fold: Given an initial cluster of documents returned from a query, construct a brief summary of the cluster. Later, given a second cluster of documents relevant to the query, generate an *update* summary, which focuses on what is new. See [1] and [2] for more details.

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[2] NIST. Text analysis conference, http://www.nist.gov/tac, 2009.

Joint work with Judith D. Schlesinger, IDA/CCS & Dianne P. O'Leary, UMCP.

On the faces of faces of the tridiagonal Birkhoff polytope

Liliana Costa, University of Aveiro, Portugal
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Thu 16:45, Room B

Doubly stochastic matrices (*i.e.* real square matrices with nonnegative entries and all rows and columns sums equal to one) have been studied quite extensively. This denomination is associated to probability distributions and it is amazing the diversity of branches of mathematics in which doubly stochastic matrices arise: geometry, combinatorics, optimization theory, graph theory and statistics. In 1946, Birkhoff published a remarkable result asserting that a matrix in the polytope of $n \times n$ nonnegative doubly stochastic matrices, Ω_n, is a vertex if and only if it is a permutation matrix. In fact, Ω_n is the convex hull of all permutation matrices of order n. The *Birkhoff polytope* Ω_n is also known as *transportation polytope* or *doubly stochastic matrices polytope.*

In 2004, Dahl, [3], discussed the subclass of Ω_n consisting of the tridiagonal doubly stochastic matrices and the corresponding subpolytope

$$\Omega_n^t = \{A \in \Omega_n : A \text{ is tridiagonal}\},$$

the so-called *tridiagonal Birkhoff polytope*, and studied the facial structure of Ω_n^t.

In this talk we present an interpretation of p-faces, $p = 0, 1, \ldots,$ of the tridiagonal Birkhoff polytope, Ω_n^t, in terms of graph theory. And, for a given p-face of Ω_n^t, we determine the number of faces of dimension zero, one, two or three, that are contained in it and we discuss their nature. In fact, a 2-face of Ω_n^t is a triangle or a quadrilateral and the 3-faces can be at most hexahedrons.

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Joint work with Enide Andrade Martins (University of Aveiro)

Matrices with Prescribed Characteristic Polynomials and Prescribed Entries

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Fri 17:10, Auditorium

An important problem that has been studied for some decades, is the description of the possible eigenvalues of a square matrix over a field, when some of its entries are prescribed and the other entries are unknown.

Another important problem that motivates our work is the description of the possible eigenvalues or the characteristic polynomial of a partitioned matrix of the form $A = [A_{i,j}]$, over a field, where the blocks $A_{i,j}$ are of type $p_i \times p_j$ $(i, j \in \{1, 2\})$, when some of the blocks $A_{i,j}$ are prescribed and the others are unknown.

In our work we intend to unify the previous problems. Indeed, our main goal is to describe the possible eigenvalues or the characteristic polynomial of a partitioned matrix of the form $C = [C_{i,j}] \in F^{n\times n}$, where F is an arbitrary field, $n = p_1 + \cdots + p_k$, the blocks $C_{i,j}$ are of type $p_i \times p_j$ $(i, j \in \{1, \ldots, k\})$, and some of its blocks are prescribed and the others vary. For this more general question we just obtained some partial results. In order to give more insight into this problem, we considered the particular situation $k = 3$.

Furthermore, we still analyze the possibility of the pair of the form

$$(C_1, C_2) = \left(\begin{bmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,1} \end{bmatrix}, \begin{bmatrix} C_{1,3} \\ C_{2,3} \end{bmatrix} \right)$$

being completely controllable (where the blocks $C_{i,j}$ are of type $p_i \times p_j$, $i \in \{1, 2\}, j \in \{1, 2, 3\}$), when three of its blocks are prescribed.

Pairs of matrices that preserve the value of a generalized matrix function on the set of the upper triangular matrices

Henrique F. da Cruz, Universidade da Beira Interior, Portugal
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Tue 15:00, Room Fermi

Let H be a subgroup of the symmetric group of degree m, let χ be an irreducible character of H and let $\mathbb{F}$ be an arbitrary field of characteristic zero. In this talk we give conditions that characterize the pairs of m-square matrices over $\mathbb{F}$, that leave invariant the value of a generalized matrix function associated with H and χ on the set of the upper triangular matrices, that is, denoting by d_χ^H the generalized matrix function associated with H and χ and by $T_n^U(\mathbb{F})$ the set of m-square upper triangular matrices, we describe the pairs (A, B) of $m \times m$ matrices over $\mathbb{F}$ that satisfies

$$d_\chi^H(AXB) = d_\chi^H(X),$$

for all $X \in T_n^U(\mathbb{F})$.

[1] Rosário Fernandes, Henrique F. da Cruz, Pairs of matrices that preserve the value of a generalized matrix function on the set of the upper triangular matrices, *submitted.*

Joint work with Rosário Fernandes (Universidade Nova de Lisboa)

An algebraic method for solving some evolution problems

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Tue 15:50, Room A

In this talk, we employ an algebraic method, which is based on resolution of linear algebraic systems, to derive traveling wave solutions for some nonlinear evolution problems. The obtained solutions include also kink solutions. Using this linear method, we present some examples which appear in various areas of applied mathematics such as modeling of fluid dynamics and population dynamics.

Imputing Missing Entries in a Data Matrix

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Wed 11:25, Room A

The problem of imputing missing entries of a data matrix is easy to state: Some entries of the matrix are unknown and we want to assign "appropriate values" to these entries. The need for solving such problems arises in several applications, ranging from traditional fields to modern ones. Typical traditional fields are Statistical analysis of incomplete survey data, Business Reports, Meteorology and Hydrology. Modern applications arise in Machine Learning, Data Mining, DNA microarrays data, Computer Vision, Recommender Systems and Collaborative Filtering. The problem is highly interesting and challenging. Many ingenious algorithms have been proposed, and there is vast literature on imputing techniques. Yet, most of the papers consider the imputing problem within the context of a specific application. The current survey attempts to provide a broader view of the problem, one that exposes the large variety of existing methods, with focus on linear algebra and optimization issues. Old and new methods are examined and explained. The equivalence theorems that we prove reveal surprising relations between apparently different methods.

The first part of the talk introduces the problem and surveys the main solution approaches. Starting from simple averaging methods we outline some basic imputing algorithms, including iterative column regression (ICR), k nearest neighbors (KNN) imputing and iterative SVD imputing. Then we move on to consider recently proposed methods, such as tail minimization (FRAA), rank minimization, and nuclear norm minimization. As our survey shows, the construction of a low-rank approximating matrix is the ultimate goal of several imputing methods. The second part of the talk considers direct minimization methods that achieve this task. The methods discussed include successive rank-one modifications (SROM), alternating least squares (ALS), Newton, Gauss-Newton, and Wiberg's algorithm.

Obtaining canonical forms associated with the problem of perturbation of one column of a controllable pair

I. de Hoyos, Universidad del País Vasco, Spain
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Fri 17:10, Room Fermi

Let (A, B) be a completely controllable matrix pair. When we consider the problem of characterizing the controllability indices of all the matrix pairs obtained by small perturbations on one column of B, a new equivalence relation arises in a natural way.

This equivalence relation is a kind of partial feedback equivalence. As a consequence the controllability indices are invariant, but they do not form a complete system of invariants.

We have found some new invariants for this equivalence relation. These invariants are of two types: continuous and discrete.

Finally, we have achieved a procedure which allows us to obtain canonical forms in terms of the invariants.

Joint work with I. Baragaña (Universidad del País Vasco), M. A. Beitia (Universidad del País Vasco)

Eigenvalues computation of possibly unsymmetric quasiseparable matrices by LR steps

Gianna M. Del Corso, Department of Computer Science, University of Pisa, Italy
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Fri 12:15, Room Pacinotti

In the last few years many numerical techniques for computing eigenvalues of structured rank matrices have been proposed. Most of them are based on QR iterations since, in the symmetric case, the rank structure is preserved and high accuracy is guaranteed. In the unsymmetric case, however the QR algorithm destroys the rank structure, which is instead preserved if LR iterations are used. We show that almost all quasiseparable matrices can be represented in terms of the parameters involved in their Neville factorization, and that this representation is preserved under LR steps. Moreover, we propose an implicit shifted LR method with a linear cost per step. We show that for totally nonnegative matrices the algorithm is stable and does not incur in breakdown also if the Laguerre shift is used. Computational evidence shows that good accuracy is obtained also when applied to symmetric positive definite matrices.

Joint work with Roberto Bevilacqua (University of Pisa) and Enrico Bozzo (University of Udine)

Preserving quasi-commutativity on self-adjoint operators

G. Dolinar, University of Ljubljana, Slovenia
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Mon 11:00, Room B

Let H be a separable Hilbert space and $\mathcal{B}_{sa}(H)$ the set of all bounded linear self-adjoint operators. We say that $A, B \in \mathcal{B}_{sa}(H)$ quasi-commute if there exists a nonzero $\xi \in \mathbb{C}$ such that $AB = \xi BA$, and that $\Phi\colon \mathcal{B}_{sa}(H) \to \mathcal{B}_{sa}(H)$ preserves quasi-commutativity in both directions when the following holds: $\Phi(A)$ quasi-commutes with $\Phi(B)$ if and only if A quasi-commutes with B. Classification of bijective maps on $\mathcal{B}_{sa}(H)$ which preserve quasi-commutativity in both directions will be presented.

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Joint work with B. Kuzma (University of Primorska)

P-rank corrections for box-constrained global optimization problems

S. Fanelli, University of Rome "Tor Vergata", Italy
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Fri 15:25, Room Galilei

In previous papers the author et alii showed that BFSG-type methods approximating the hessian of twice continuously differentiable functions with a structured matrix are very efficient to compute local minima, particularly in the secant case. Moreover, by utilizing a suitable BFGS-type algorithm, a general theorem ensuring the convergence to the global minimum of unconstrained twice continuously differentiable functions was recently proved.
A family of important deterministic methods for global optimization is based upon the theory of terminal attractors and repellers. Unfortunately, the utilization of scalar repellers is unsuitable when the dimension n of the problem assumes values of operational interest.
On the other hand, the algorithms founded on the classical αBB technique are often ineffective for computational reasons, even if, more recently, the utilization of a new class of convex under-estimators and relaxations has significantly improved the performances of this approach.
In order to increase the power of the repeller in the tunneling phase, the utilization of repeller matrices with a proper structure is certainly promising and deserves investigation. More precisely, it is interesting to test the performances obtained by approximating the optimal (unknown) repeller matrix with the sum of a diagonal matrix and a low rank one. The corresponding tunneling phase must be, in fact, properly superimposed in the global optimization algorithm in the frame of the αBB computational scheme.
Numerical experiences on a wide set of classical and well known optimization problems show that the latter approach has a significant effect on the efficiency of the whole global optimization procedure.

Commutativity preservers on matrix algebras

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Tue 12:15, Room B

Let $M_n(\mathbb{F})$ be the algebra of all $n \times n$ matrices over the field $\mathbb{F}$. A map $\phi : M_n(\mathbb{F}) \to M_n(\mathbb{F})$ preserves commutativity if $\phi(A)\phi(B) = \phi(B)\phi(A)$ whenever $AB = BA$, $A, B \in M_n(\mathbb{F})$. If ϕ is bijective and both ϕ and ϕ^{-1} preserve commutativity, then we say that ϕ preserves commutativity in both directions. We will represent recent results on general (non-linear) maps on some matrix algebras that preserve commutativity in both directions or in one direction only. We will talk about complex and real matrices, hermitian, symmetric, and alternate matrices.

On J-normal matrices with J'-normal principal submatrices

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Tue 12:15, Room A

Let M_n be the algebra of $n \times n$ complex matrices and let $J = I_r \oplus -I_{n-r} \in M_n$, $0 \leq r \leq n$. Consider the indefinite inner product $[\cdot,\cdot]$ defined by $[x, y] = y^*Jx$, $x, y \in \mathbb{C}^n$. A matrix $A \in M_n$ is said to be J-normal if $A^\# A = AA^\#$, in which $A^\#$ is the J-adjoint of A defined by $[Ax, y] = [x, A^\# y]$ for any $x, y \in \mathbb{C}^n$ (that is, $A^\# = JA^*J$).

A matrix B of size $m \times m$, $m < n$, is said to be imbeddable in $A \in M_n$ if there exists a matrix V of size $n \times m$ such that $V^\# V = I_m$ and $V^\# AV = B$.

Let $J' = I_{r-p} \oplus -I_{n-r-q}$, with $0 \leq p \leq r$, $0 \leq q \leq n - r$. In this talk we consider the following problem: give necessary and sufficient conditions for a J'-normal matrix $B \in M_{n-p-q}$ to be imbeddable in a J-normal matrix $A \in M_n$. We present an answer to this problem in some particular cases. When $n = r$, the matrix A is normal and the problem was solved by Fan and Pall (1957) for $q = 1$. When $n = r$ and A has real eigenvalues, then A is Hermitian and the answer to the problem is given by the well known interlacing relations for the eigenvalues of A and B.

Joint work with N. Bebiano and J. Providencia (University of Coimbra)

A refined Young inequality and related results

S. Furuichi, Nihon University, Tokyo, Japan
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Mon 17:35, Room C

In this talk, we study on refinements of some inequalities related to Young inequality for scalar and for operator. As our main results, we show refined Young inequalities for two positive operators. Our results refine the ordering relations among the arithmetic mean, the geometric mean and the harmonic mean. Moreover, we give supplements for refined Young inequalities for two positive real numbers. And then we also give operator inequalities based on the supplemental inequalities.

In addition, (if we have an enough time to talk), we show two type of the reverse inequalities of the refined Young inequality for two positive operators, applying the reverse inequalities of the refined Young inequality for positive real numbers

Our talk is based on our recent results [1,2].

[1] S. Furuichi and M. Lin, On refined Young inequalities, arXiv:1001.0195.
[2] S. Furuichi, Reverse inequalities for a refined Young inequality, arXiv:1001.0535.

Joint work with Minghua Lin (University of Regina)

Disturbance decoupling for singular systems by feedback and output injection

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Tue 15:00, Room A

We study the disturbance decoupling problem for linear time invariant singular systems. We give necessary and sufficient conditions for the existence of a solution to the disturbance decoupling problem with or without stability via a proportional and derivative feedback and proportional and derivative output injection that also makes the resulting closed-loop system regular and/or of index at most one. All results are based on canonical reduced forms that can be computed using a complete system of invariants.

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Full Rank Factorization with Quasy Neville Elimination Process

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Thu 15:25, Room Galilei

Let A be a real $m\times n$ matrix with $rank(A) = p$. A decomposition $A = LS$ is called full rank factorization of A, if $L \in R^{m\times p}$, $S \in R^{p\times n}$ and $rank(L) = rank(S) = p$. Several authors have studied different classes of matrices obtaining properties and characterizations of them in terms of full rank factorizations. Recently, Cantó et al. (see [1]) obtain a characterization of *tn* (totally negative) and *tnp* (totally nonpositive) matrices in terms of their full rank factorization in echelon form.

In [2] the authors introduced a variant of the Neville elimination process, *Quasi-Neville elimination*, which consists of leaving the zero row in its position and continuing the elimination process with the matrix obtained from A by deleting the zero rows. The essence of this process is to use the property N introduced by M. Gasca and J.M. Peña in [3]: *An $n \times m$ real matrix A satisfies the condition N if whenever we have carried some rows down to the bottom in the Neville elimination of A, those rows were zero rows, and the same condition is satisfied in the Neville elimination of U^T.* In this work we introduce a new class of matrices weakening the N condition.

Definition. Let A be an $m\times n$ real matrix. A satisfies the condition WN if A satisfies the property N only for rows.

By applying Quasy Neville elimination process we can prove the following result.

Theorem. Let A be an $m\times n$ real matrix, with $rank(A) = p$ and satisfying the WN condition. Then A admits a full rank factorization in echelon form

$$A = LS,$$

where L is a lower echelon matrix of size $m \times p$, S is an upper echelon matrix of size $p \times n$ and $rank(L) = rank(S) = p$.

From this result we obtain a full rank factorization in echelon form of a class of matrices that contains the classes of *tn*, *tnp*, TP (totally positive), TNN (totally nonnegative) matrices. This class also includes the sign regular matrices introduced in [4], some Vandermonde matrices and the semiseparable matrices.

Consider the following matrix

$$A = \begin{bmatrix} 2 & 3 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 2 & 4 & 5 \\ 1 & 3 & 2 & 4 & 6 \end{bmatrix}.$$

We can prove that this matrix satisfies the WN condition, but does not satisfy the N condition. In addition, we can observe that this matrix is neither TP, TNN, *tn* nor *tnp*.

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Joint work with M. Abad and Juan R. Torregrosa (Instituto de Matemática Multidisciplinar, Universidad Politécnica de Valencia, Spain)

The Padé iterations for the matrix sign function and their reciprocals are optimal

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Tue 15:25, Room Fermi

Rational iterations of the form $z_{k+1} = \varphi(z_k)$, for some rational function $\varphi(z) = a(z)/b(z)$, having attractive fixed points at 1 and -1 locally converge to the sign function and thus they can be used to compute important matrix functions such as the matrix sign function and the matrix square root.

We show that among the rational iterations locally converging with order $s > 1$ to the sign function, the ones belonging to the Padé family and their reciprocals are the unique with the lowest sum of the degrees of numerator and denominator.

This provides a good motivation for their choice in numerical computation.

Joint work with B. Iannazzo (Università di Perugia) and F. Poloni (SNS, Pisa)

Perturbation theory for block operator matrices and applications

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Thu 11:50, Room A

Block operator matrices are matrices whose entries are linear operators on Hilbert or Banach spaces. Both bounded and unbounded operators are allowed as matrix entries. Such objects have found various applications—over the last 20 years—in both applied as well as theoretical mathematics. However, until an excellent recent monograph of C. Tretter their spectral theory has not found its way into standard textbooks. We refer an interested reader to the monograph for a extensive review of the relevant literature. In this talk we present new results in the relative perturbation theory for unbounded block operator matrices. As a first application we discuss adaptive finite element eigenvalue methods from the viewpoint of the spectral theory of block operator matrices. As a result we obtain robust reliability and efficiency estimates for the eigenvalue and eigenvector estimation. Furthermore, we introduce the notion of the enhanced Ritz value and show that it can be used to obtain new computable eigenvalue enclosures which are sharper than those which can be obtained from Ritz values. To illustrate the versatility of the block operator matrix approach we briefly and informally show an application of our results in the spectral theory of operator realizations of elliptic systems of partial differential equations. The basic flavor of the whole theory is a notrivial application of standard results from Linear Algebra in the rigorous theory of elliptic partial differential equations.

A Null Space Free Jacobi-Davidson Iteration for Three Dimensional Photonic Crystals

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Thu 15:50, Room C

We present an efficient null space free Jacobi-Davidson method to compute the positive eigenvalues of the degenerate elliptic operator arising from Maxwell's equations. We consider spatial compatible discretizations such as Yee's scheme which guarantee the existence of a discrete vector potential. During the Jacobi-Davidson iteration, the correction process is applied to the vector potential instead. The correction equation is solved approximately as in standard Jacobi-Davidson approach. The computational cost of the transformation from the vector potential to the corrector is negligible. As a consequence, the expanding subspace automatically stays out of the null space and no extra projection step is needed. This new method is mathematically equivalent to the standard Jacobi-Davidson method for solving the corresponding generalized eigenvalue problem but the expanding subspace automatically stays out of the null space. Numerical evidence confirms that the new method is much more efficient than the standard Jacobi-Davidson method.

Joint work with Yin-Liang Huang (National Taiwan University), Wen-Wei Lin (National Chiao Tung University) and Wei-Cheng Wang (National Tsing Hua University)

***H*-expansive matrices in indefinite inner product spaces and their invariant subspaces**

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Tue 11:00, Room A

We consider indefinite inner products given by a square real invertible symmetric matrix $H = H^T : [x, y] = (Hx, y)$, [1]. On the Euclidean space equipped with this indefinite inner product, we consider matrices A for which $A^*HA - H$ is nonnegative definite. Such matrices are called H-expansive matrices.

We are interested in the construction of A-invariant maximal H-nonnegative and nonpositive subspaces. The complex case has already been treated by means of a suitable Cayley transform, [2]. The problem when A is real and A has both 1 and -1 as eigenvalues cannot be treated in a straightforward way by means of Cayley transform. We propose a more direct approach. The uniqueness and stability of these subspaces are also studied.

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Joint work with J.H. Fourie (NWU, Potchefstroom, SA), G. Groenewald (NWU, Potchefstroom, SA), A.C.M. Ran (VU, Amsterdam, NL)

The Development of Excel and Sage Math tools for Linear Algebra

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Fri 15:25, Room A

It has been well-known that applications of technology is getting more important for our Linear Algebra class. In particular, MS Excel and Sage Math have been a powerful tools on E-learning environment of today. We will introduce what we have done on the developement of MS Excel tools and Sage Math tools for our Linear Algebra class. We would like share our experiences in this talk.

On a confluent Vandermonde matrix polynomial

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Mon 15:50, Room A

In a paper, [1], the null space of a Vandermonde matrix polynomial of block Toeplitz type has been studied. This was of relevance for the characterization of a matrix polynomial equation having non-unique solutions. The origin of the problem can be retrieved in [2], where an interconnection between the Fisher information matrix of an ARMAX process and a solution to a Stein equation is established. We shall now consider the problem by embedding the results in [1] in a much more general approach to obtain properties of matrix polynomials that can be viewed as generalizations of a confluent Vandermonde matrix.

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[2] A.Klein and P.Spreij, On the solution of Stein's equation and Fisher's information matrix of an ARMAX process, Linear Algebra and its Applications, 396 (2005), 1–34.

Joint work with Peter Spreij (University of Amsterdam)

Solvability of regular pencils for quadratic inverse eigenvalue problem

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Fri 17:35, Auditorium

In this paper, we are interested in the study of solvability of the quadratic inverse eigenvalue problem (QIEP) of dimension n. Let $k_* = (1 + \sqrt{1 + 8n})/2$ and $0 \le k < k_*$, and for $m := n + k$ prescribed eigenpairs $\{(\lambda_j, x_j)\}_{j=1}^m$, we prove that, generically, there is a constructible nonsingular symmetric quadratic pencil solution $Q(\lambda) \equiv \lambda^2 M + \lambda C + K$ to the QIEP such that $Q(\lambda_j)x_j = \mathbf{0}$ $(j = 1, \ldots, m)$. If $k_* \le k \le n$, we show that, generically, all symmetric quadratic pencil solutions are singular. We also derive the dimension of the solution subspace of the QIEP for both cases. Furthermore, we develop an algorithm for finding a symmetric positive definite M for the QIEP if it exists.

Joint work with Yunfeng Cai (Peking University), Wen-Wei Lin (National Chiao Tung University) and Shu-Fang Xu (Peking University)

Jordan orthogonality homomorphisms on Hermitian matrices.

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Mon 12:15, Room B

One of the products to consider on complex Hermitian matrices is the Jordan product $AB + BA$. We say that two Hermitian matrices are Jordan-orthogonal if their Jordan product vanishes. Additive maps which preserve Jordan orthogonality on Hermitian matrices and their infinite-dimensional counterpart, i.e., self-adjoint operators, have recently been investigated by Hou and Zhao [1] on self-adjoint operators and by Chebotar, Ke and Lee [2] on matrix rings with involution.

In applications it is imperative to obtain strong structural results with the minimum possible assumptions. We thereby classify homomorphisms of Jordan orthogonality on Hermitian matrices, i.e. we classify maps (without additivity or bijectivity assumptions) with the property that $AB+BA = 0$ implies the same condition on the images of matrices. We also added one rather small, but unavoidable, technical assumption that no nonzero matrix is annihilated. With such a limited restrictions on map we can not hope for a nice structural result on the whole Hermitian matrices. Nonetheless, with the help of our results we could show that every nonconstant homomorphism of Jordan product on Hermitian matrices (again nor additivity nor bijectivity is assumed) is automatically a linear Jordan isomorphism.

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Joint work with A. Fošner (IMFM Slovenia), N.-S. Sze (Dept. of Applied Math., The Hong Kong Polytechnic University, Humg Hom, Hong Kong), and T. Kuzma

Trace Inequalities for Logarithms and Powers of J-Hermitian Matrices

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Mon 17:10, Room C

Some spectral inequalities are presented for the trace of logarithms, exponentials and powers of J-Hermitian matrices, $J = I_r \oplus -I_{n-r}, 0 < r < n$. These inequalities are established in the context of indefinite inner product spaces and they are known to be valid for Hilbert space operators or operator algebras.

Key words: Indefinite inner product space, J-Hermitian matrix, relative entropy, Tsallis entropy, Klein inequality, Peierls-Bogoliubov inequality.

Joint work with N. Bebiano (University of Coimbra), J. Providência (University of Coimbra), G. Soares (University of Trás-os-Montes e Alto Douro)

Integer partitions and linear systems over the ring of real continuous functions defined on the unit circle

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Thu 15:00, Room B

A locally Brunovsky linear system is a reachable linear system locally feedback equivalent to a classical canonical form.
Let R be the ring of real continuous functions defined on the unit circle. We describe an algorithm for generating all locally Brunovsky classes over R and give a bound of the number of such classes through integer partitions with special conditions.

Generalized Krein Conditions on the Parameters of a Strongly Regular Graph

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Thu 15:50, Room A

Let X be a strongly regular graph with three distinct eigenvalues. We associate a three dimensional Euclidean Jordan algebra V to the adjacency matrix of X. Then we generalize the Krein parameters of a strongly regular graph and obtain some generalized Krein admissibility conditions for strongly regular graphs.

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Joint work with Domingos Cardoso (University of Aveiro, CEOC), Enide Martins (University of Aveiro, CEOC), Luis Vieira (University of Porto, CMUP)

Accurate eigenvalues of Said-Ball-Vandermonde matrices

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Fri 11:00, Room Pacinotti

Said-Ball-Vandermonde matrices are a generalization of the Vandermonde matrices arising when the power basis is replaced by the Said-Ball basis. When the nodes are inside the interval $(0, 1)$, then those matrices are strictly totally positive [1]. In this work an algorithm for computing the bidiagonal decomposition of those Said-Ball-Vandermonde matrices is presented, which allows us to use known algorithms for computing the eigenvalues of totally positive matrices represented by their bidiagonal decomposition [2]. The algorithm is shown to be fast and to guarantee high relative accuracy. Some numerical experiments which illustrate the good behaviour of the algorithm are included.

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Joint work with J. J. Martínez (University of Alcalá)

Multi-way adaptive solution of parametric PDE eigenvalue problems

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Mon 15:00, Room Fermi

In this talk we present a multi-way adaptive method for parametric eigenvalue problems. A posteriori error estimates for eigenvalues and associated eigenfunctions for both self- and non-selfadjoint problems will be introduced. These estimates take into account an inexact solution of the corresponding algebraic eigenvalue problem. In our adaptive algorithm we balance the discretization and algebraic error, and introduce the efficient stopping criteria. Additionally, for the non-selfadjoint problem we discuss a new computational procedure based on the adaptive homotopy approach. This is partially a joint work with C. Carstensen, J. Gedicke (HU Berlin, Germany).

Joint work with Volker Mehrmann (TU Berlin, Germany)

On the growth factor for generalised orthogonal matrices

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Mon 11:00, Room A

When Gaussian elimination is applied on a completely pivoted (CP) matrix A the growth factor is defined as $g(n,A) = \frac{\max\{p_1,p_2,\ldots,p_n\}}{|a_{11}|}$, where $p_1, p_2, \ldots, p_n$ are the pivots of A. In 1968 Cryer [1] formulated the following conjecture.

$g(n,A) \leq n$, with equality iff A is a Hadamard matrix.

We will describe the progress on the equality part of this conjecture by presenting all the results concerning the growth factor for Hadamard matrices H_n of dimension n, for binary Hadamard matrices and for weighing matrices of order n and weight k. The latest matrices can achieve moderate growth factor. All these matrices are special cases of generalised orthogonal matrices. Also we will develop theoretical methodologies [3,4] computing the minors of the above type matrices which can lead to numerical algorithms evaluating their pivots. A great difficulty arises at the study of this problem because the pivot pattern is not invariant under H-equivalence. In [2] the unique pivot pattern for H_{12} was presented and in [5] all 34 possible pivot patterns of H_{16} were demonstrated theoretically and the complete pivoting conjecture for H_{16} was proved. The determination of the pivot patterns for H_{20} and for higher dimensions remains open.

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[4] C. Kravvaritis and M. Mitrouli, Evaluation of Minors associated to weighing matrices, Linear Algebra Appl. 426 pp. 774-809, 2007.
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High Relative Accuracy Implicit Jacobi Algorithm for the SVD

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Mon 15:25, Room Fermi

We prove that a Jacobi-like algorithm applied implicitly on a decomposition $A = XDY^T$ of a matrix A, where D is diagonal, and X, Y are well conditioned, computes all singular values of A to high relative accuracy. The relative error in every eigenvalue is bounded by $O(\epsilon \max[\kappa(X), \kappa(Y)])$, where ϵ is the machine precision and $\kappa(X) = \|X\|_2\|X^{-1}\|_2, \kappa(Y) = \|Y\|_2\|Y^{-1}\|_2$ are, respectively, the spectral condition number of X and Y. The singular vectors are also computed accurately in the appropriate sense. We compare it with previous algorithms for the same problem [1] and see that the new algorithm is faster and more accurate. This work is an extension of the Jacobi implicit algorithm presented in [2] for the symmetric eigenproblem.

[1] J. Demmel et al., Linear Algebra and its Applications, 299 (1999) 21-80
[2] F. M. Dopico, P. Koev and J. M. Molera, Numer. Math. 113 (2009) 519-553

Joint work with Froilán M. Dopico and Johan Ceballos (Universidad Carlos III de Madrid, Spain)

Tensor approach to mixed high-order moments of absorbing Markov chains

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Thu 15:50, Room B

Moments of an absorbing Markov chain are considered. First moments and non-mixed second moments of the number of visits are determined in classical textbooks such as the book of J. Kemeny and J. Snell “Finite Markov Chains”. The reason is that the first moments and the non-mixed second moments can be easily expressed in a matrix form using the fundamental matrix of the absorbing Markov chain. Since the representation of the mixed moments of higher orders in a matrix form is not straightforward, if ever possible, they were not calculated. The gap is filled now. Tensor approach to the mixed high-order moments is proposed and compact closed-form expressions for the moments are discovered.

Computing Low Rank Approximations of Tensors

Mechie Nkengla, University of Illinois Chicago, USA
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Mon 12:15, Room A

We work on reliable and efficient algorithms for the best low rank approximation and decompositions of tensors. By exploring non-orthogonality required decompositions such as a CUR-like method for tensor, we investigate the concept of whether unfolding (or matricization) in a particular mode makes a difference in the approximation scheme by experimentation on very large data sets. We also show that by padding the large data set with zeroes, the computational cost of the least-square algorithm is improved. Heuristic based methods such as this provide an advantage in terms of computational complexities and we show that even without an intrinsic bound on the approximation error, the approximations are quite acceptable.

Joint work with Shmuel Friedland (University of Illinois at Chicago)

Exploiting structures in palindromic polynomial eigenvalue problems

Vanni Noferini, University of Pisa, Italy
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Fri 17:35, Room Fermi

Representing palindromic matrix polynomials in the Dickson polynomial basis leads to structured generalized eigenvalue problems. We propose a linearization of the latter problem by means of a suitable matrix pencil having rank-structured block coefficients. We discuss a stragegy, based on the QZ method, to exploit this rank-structure and to tackle the associated nonlinear eigenvalue problem.

Commuting and noncommuting graphs of matrices over semirings
Polona Oblak, University of Ljubljana, Slovenia
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Thu 17:35, Room C

The *commuting graph* $\Gamma(S)$ of a set S is the graph, whose vertex set is the set of all noncentral elements of S and $x - y$ is an edge in $\Gamma(S)$ if $xy = yx$ and $x \neq y$. Its complement is called the *noncommuting graph* of S.

In the talk, we give diameters and girths of the commuting and noncommuting graphs of certain subsets of matrices over semirings, namely for the set of nilpotent matrices, invertible matrices, noninvertible matrices and the full matrix semiring.

Joint work with David Dolžan (University of Ljubljana)

PageRank and Social Competences on Social Network Sites
F. Pedroche, Institut de Matemàtica Multidisciplinària. Universitat Politècnica de València. Espanya.
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Wed 12:15, Room A

In this communication a new method to classify the users of an SNS (Social Network Site) into groups is shown. The method is based on the PageRank algorithm. *Competitivity groups* are sets of nodes that compete among each other to gain PageRank via the *personalization vector*. Specific features of the SNSs (such as number of friends or activity of the users) can modify the ranking inside each *Competitivity group*. We call these features *Social Competences*. Some numerical examples are shown.

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Computation of Canonical Forms and Miniversal Deformations of Bimodal Dynamical Systems
M. Peña, Universitat Politècnica de Catalunya, Spain
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Tue 17:35, Room B

Canonical forms for controllable bimodal dynamical linear systerms (BDLS) have been used by different authors. The uncontrollable case appears naturally, for example, when considering parametrized families of such systems, where the uncontrollability of some of their members can not be avoided by means of a generic perturbation. Here we provide an algorithm to obtain canonical forms for BDLS, both in the controllable and uncontrollable cases, for $n = 2$ and $n = 3$, which are the most frequent dimensions in the applications. We apply them to compute the miniversal deformation of a triple defining a BDLS, in order to study its local perturbations and bifurcation diagram.

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Joint work with J. Ferrer, M. D. Magret, J. R. Pacha (Universitat Politècnica de Catalunya)

On the spectral radius of non-negative matrices
A. Peperko, University of Ljubljana, Slovenia
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Mon 15:25, Room B

Let $K_1, \ldots, K_n$ be (infinite) non-negative matrices that define operators on a Banach sequence space. Given a function $f : [0,\infty) \times \ldots \times [0,\infty) \to [0,\infty)$ of n variables, we define a non-negative matrix $\hat{f}(K_1, \ldots, K_n)$ and consider the inequality

$$r(\hat{f}(K_1, \ldots, K_n)) \leq \frac{1}{n}\left(r(K_1) + \cdots + r(K_n)\right),$$

where r denotes the spectral radius. We find the largest function f for which this inequality holds for all $K_1, \ldots, K_n$. We also obtain an infinite-dimensional extension of the result of Cohen asserting that the spectral radius is a convex function of the diagonal entries of a non-negative matrix.

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Joint work with R. Drnovšek (University of Ljubljana, Slovenia)

Singular two-parameter eigenvalue problems and bivariate polynomial systems

B. Plestenjak, University of Ljubljana, Slovenia
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Mon 17:10, Room Fermi

It is well known that roots of a scalar polynomial $p(x)$ are the eigenvalues of its companion matrix. Therefore, one can apply various numerical methods for the eigenproblem to compute the roots of the polynomial.

We will generalize this approach to bivariate polynomial systems

$$\begin{aligned} p_1(x,y) &= 0, \\ p_2(x,y) &= 0. \end{aligned} \tag{1}$$

It is possible to construct matrices A_i, B_i, and C_i, such that $\det(A_i + \lambda B_i + \mu C_i) = p_i(\lambda, \mu)$ for $i = 1, 2$. The roots of (1) are then the eigenvalues of the *two-parameter eigenvalue problem*

$$\begin{aligned} A_1 x_1 &= \lambda B_1 x_1 + \mu C_1 x_1, \\ A_2 x_2 &= \lambda B_2 x_2 + \mu C_2 x_2. \end{aligned} \tag{2}$$

The dimension of the matrices A_i, B_i and C_i is much larger than the order of the polynomial p_i and the two-parameter eigenvalue problem (2) is singular. Recent results and numerical methods for singular two-parameter eigenvalue problems [2,3] enable us to compute the finite eigenvalues of (2). Combined with the Jacobi–Davidson approach [1], this might be an alternative when we are interested only in part of the roots that are close to a given target.

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Joint work with A. Muhič (University of Ljubljana)

When several matrices share an invariant cone ?

V.Yu. Protasov, Moscow State University, Russia
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Mon 15:00, Room B

We analyze finite families of linear operators $\{A_1, \ldots, A_m\}$ acting in $\mathbb{R}^d$ and sharing a common invariant cone in that space, i.e., there is a closed pointed nondegenerate cone $K \subset \mathbb{R}^d$ such that $A_i K \subset K$ for all $i = 1, \ldots, m$. Special properties of such families have found many applications in the study of joint spectral radii, the Lyapunov exponents, combinatorics, graphs and large networks, etc. Operators with a common invariant cone, act, in some sense, "in the same direction" and inherit most of special properties of matrices with nonnegative entries. We preset a sharp criterion for a finite family of operators to possess a common invariant cone. The criterion reduces the problem to equality of two special numbers that depend on the family. In spite of theoretical simplicity of the criterion, the practical implementation may be difficult because of the high algorithmic complexity of the problem. We show that the problem of existence of a common invariant cone for four matrices with integral entries is algorithmically undecidable. This means that there is no algorithm, which for any family of four matrices with integral entries gives the answer "yes" or "no" within finite time. In particular, this problem is NP-hard. On the other hand, some corollaries of the criterion lead to simple sufficient and necessary conditions for the existence of an invariant cone. Finally, we formulate an approximative analogue of the problem and introduce a "co-directional number" of several matrices. This parameter is close to zero if and only if there is a small perturbation of matrices, after which they get an invariant cone. An algorithm for its computation is presented.

The distance from a matrix polynomial to a prescribed multiple eigenvalue

P. Psarrakos, National Technical University of Athens, Greece
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Thu 12:15, Room A

The *spectrum* of an $n \times n$ *matrix polynomial* $P(\lambda) = \sum_{j=0}^{m} A_j \lambda^j$ ($\det A_m \neq 0$) is $\sigma(P) = \{\lambda \in \mathbb{C} : \det P(\lambda) = 0\}$. An *eigenvalue* $\lambda_0 \in \sigma(P)$ is called *multiple* if its multiplicity as a zero of $\det P(\lambda)$, that is, its *algebraic multiplicity*, is greater than 1. Moreover, the *geometric multiplicity of* $\lambda_0 \in \sigma(P)$ is the dimension of the null space of matrix $P(\lambda_0)$. In this work, we are interested in perturbations of $P(\lambda)$ of the form $Q(\lambda) = \sum_{j=0}^{m} (A_j + \Delta_j)\lambda^j$, where the matrices $\Delta_j \in \mathbb{C}^{n\times n}$ are arbitrary. In particular, for a scalar $\mu \in \mathbb{C}$, we define a distance from $P(\lambda)$ to μ as a multiple eigenvalue and a distance from $P(\lambda)$ to μ as an eigenvalue with geometric multiplicity κ. Using the singular value decomposition of matrix $P(\mu)$, we compute the first distance and an associated optimal perturbation of $P(\lambda)$. Moreover, for the second distance, we obtain upper and lower bounds, constructing perturbations of $P(\lambda)$ that correspond to the upper bounds. Finally, numerical examples are presented to illustrate and evaluate our results.

Joint work with N. Papathanasiou (National Technical University of Athens)

Minimum polynomials and spaces of matrices with special rank properties

Rachel Quinlan, National University of Ireland, Galway
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Mon 11:50, Room B

Let V be a vector space of dimension n over a field K that admits cyclic extensions of degree n. Then V may be equipped with the structure of a field extension L of K, with cyclic Galois group $\langle\sigma\rangle$ of order n. From Artin's theorem on linear independence of characters it follows that every K-linear endomorphism of V has a unique expression of the form

$$a_0 \mathrm{id} + a_1\sigma + \cdots + a_{n-1}\sigma^{n-1}, \quad a_i \in L.$$

Thus the K-linear endomorphisms of L can be identified with "polynomial-type" expressions of degree at most $n-1$ in σ. If $p(\sigma)$ is such an expression, we show that the kernel of the endomorphism corresponding to $p(\sigma)$ is at most equal to the degree of $p(\sigma)$. Furthermore, if $W = \langle a_1, a_2, \ldots a_k\rangle$, we show that up to multiplication by an element of $L^\times$ there is a unique polynomial of degree k in σ that annihilates exactly W. Such a polynomial is given by

$$m_W(\sigma) = \det \begin{pmatrix} a_1 & a_2 & \dots & a_k & \mathrm{id} \\ \sigma(a_1) & \sigma(a_2) & \dots & \sigma(a_k) & \sigma \\ \vdots & \vdots & & \vdots & \vdots \\ \sigma^k(a_1) & \sigma^k(a_2) & \dots & \sigma^k(a_k) & \sigma^k \end{pmatrix}$$

By considering K-endomorphisms of V as L-linear combinations of Galois group elements, it is possible to construct subspaces with special rank properties of the space $M_n(K)$ of $n \times n$ matrices over K, and its subspaces $A_n(k)$ of skew-symmetric matrices and $S_n(k)$ of symmetric matrices. For example if $n = 2m+1$ is odd, it can be shown that $A_n(K)$ contains a chain of subspaces

$$A_n(k) = A_0 \supset A_1 \supset A_2 \supset \cdots \supset A_m = 0.$$

with the property that A_{i+1} has codimension n in A_i and for each $i < m$ the non-zero elements of A_i all have rank exceeding $2i$.

Joint work with R. Gow (University College Dublin)

Integral graphs with regularity constraints

P. Rama, University of Aveiro, Portugal
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Fri 16:45, Room C

Given a graph $G = (V(G), E(G))$, a subset of vertices $\emptyset \neq S \subseteq V(G)$ is a (k,τ)-regular set if S induces a k-regular subgraph in G and every vertex $v \in V(G) \setminus S$ has exactly τ neighbors in S.
We characterize some known classes of integral graphs with (k,τ)-regular sets corresponding to all distinct eigenvalues and identify some particular integral graphs with this property. We also present some graph operations that generate integral graphs with (k,τ)-regular sets for all distinct eigenvalues from integral graphs with the same property.

Joint work with P. Carvalho (University of Aveiro)

The pair of operators $T^{[*]}T$ and $TT^{[*]}$; J–dilations and canonical forms.

A.C.M. Ran, Department of Mathematics, VU University Amsterdam
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Tue 11:25, Room A

The problem of comparing the operators $T^{[*]}T$ and $TT^{[*]}$ in indefinite inner product spaces has already attracted some attention. One of the motivations was a result stating that a matrix T admits polar decomposition if and only if the canonical forms of $T^{[*]}T$ and $TT^{[*]}$ are the same. In the finite dimensional situation canonical forms of the matrices in question were considered for some special cases in [4]. Later on in [1] a full description was provided. On the other hand, the infinite dimensional case is far from being fully understood. For example, zero can be a singular critical point of one of the operators, while it is in the positive spectrum of the other operator. Further examples can be found in [2], where the notions of regular and singular critical point were studied for the pair $T^{[*]}T$ and $TT^{[*]}$. In this talk we present a method of dilation (reduction) for the operator T, which is quite natural for the study of the properties of $T^{[*]}T$ and $TT^{[*]}$. This construction has its origins in [3], and is similar to a construction implicitly used in [1]. Both the infinite and the finite dimensional case will be discussed, as well as an alternative proof of one of the main results of [1].

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Joint work with M. Wojtylak

On the pole placement problem for singular systems

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Tue 16:45, Room B

Given a singular system with outputs

$$\begin{aligned} E\dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned}$$

$E, A \in F^{h\times n}, B \in F^{h\times m}, C \in F^{p\times n}$, and a monic homogeneous polynomial $f \in F[x,y]$, we obtain necessary and sufficient conditions for the existence of a state feedback matrix F and an output injection K such that the state matrix $sE-(A+BF+KC)$ has f as characteristic polynomial, under a regularizability condition on the system.

Joint work with F.C. Silva (University of Lisbon, Portugal)

Partitioned triangular tridiagonalization: rounding error analysis

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Mon 15:50, Room Fermi

We consider a partitioned algorithm for reducing the symmetric matrix A to tridiagonal form, which computes a factorization $PAP^T = LTL^T$ where P is a permutation matrix, L is lower triangular with a unit diagonal and bounded off-diagonal elements, and T is symmetric tridiagonal. We show that such a partitioned factorization is backward stable provided that the corresponding growth factor is not too large (the entries can grow in the factor T). The only slight change with respect to the basic (nonpartitioned) algorithm is in the constant that includes the the size of partition which, on the other hand, allows to exploit modern computer architectures through the use of the level-3 BLAS. Experimental results demonstrate that such algorithm achieves approximately the same level of performance as the blocked Bunch-Kaufman code implemented in Lapack. The Bunch-Kaufman method is also conditionally backward stable (assuming no or moderate growth in triangular factors) making these two main approaches comparable also from the numerical stability point of view.

[1] M. Rozloznik, G. Shklarski and S. Toledo: Partitioned triangular tridiagonalization, to appear in ACM Transactions

on Mathematical Software.

Joint work with G. Shklarski and S. Toledo (Tel-Aviv University)

Single-input systems over von Neumann regular rings

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Thu 15:50, Room Galilei

This talk deals with the study of linear systems with scalars in a commutative von Neumann regular ring, i.e. a zero-dimensional ring with no nonzero nilpotents (for example $\mathbb{Z}/(n)$, where n is a squarefree integer). It is shown that a commutative ring is von Neumann regular if and only if any single-input system is feedback equivalent to a special normal form. This normal form, which can be obtained by an explicit algorithm, is associated to a collection of principal ideals which determine completeley the structure of the reachability submodule of the system.

K-hyperbolas and polynomial numerical hulls of normal matrices

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Fri 11:50, Room Galilei

Let $A \in M_n$ be a normal matrix and let $k \in \mathbb{N}$. In this note we introduce the notions "k-hyperbola" and "k-hyperbolic region". The polynomial numerical hull of order k, denoted by $V^k(A)$ is characterized by the intersection of k-hyperbolic regions. Also, the locus of $V^{n-1}(A)$ in the complex plane is determined.

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Joint work with Hamid Reza Afshin, Mohammad Ali Mehrjoofard (Vali-E-Asr University of Rafsanjan, Rafsanjan, Iran)

Spectra and cycles of length m in regular tournaments of order n

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Fri 17:10, Room C

A tournament T is an orientation of a complete graph. A tournament is regular if the out-set and in-set of each vertex have the same number of vertices. Let $c_m(T)$ be the number of cycles of length $m \geq 3$ in T. It is well known that any two regular tournaments of (odd) order n have the same number of cycles of length 3 (M.G. Kendall & B. Babington Smith, 1940). The case of $m = 4$ is more complicated. Let RLT_n be the unique regular locally transitive tournament of order n and DR_n be a doubly-regular tournament of this order. According to the results of U. Colombo (1964) and B. Alspach & C. Tabib (1982), for any regular tournament T of order n, we have $c_4(DR_n) \leq c_4(T) \leq c_4(RLT_n)$. In the present talk, based on matrix methods, we show that $2c_4(T) + c_5(T) = n(n-1)(n+1)(n-3)(n+3)/160$. This implies $c_5(RLT_n) \leq c_5(T) \leq c_5(DR_n)$ for any regular tournament T of order n.

Note that if $m = 3, 4, 5$, then $mc_m(T)$ is equal to the trace $tr_m(T)$ of the mth power of the adjacency matrix of T. While $6c_6(T)$ does not equal $tr_6(T)$, we show that $c_6(T)$ is also a function of the spectrum of a regular tournament T. The corresponding pure spectral expression for $c_6(T)$ allows us to prove the inequality $c_6(T) \leq c_6(DR_n)$ with equality holding if and only if $T = DR_n$ or $T = Kz_7$. We also determine the value of $c_6(RLT_n)$ and conjecture that this value is the minimum number of 6-cycles in the class of regular tournaments of order n. Finally, we determine the coefficients at n^m and n^{m-1} in the expansion of the expression for both $c_m(DR_n)$ and $c_m(RLT_n)$ in the case of arbitrary length $m > 3$. This allows us to state that for a sufficiently large order n, $c_m(RLT_n) < c_m(DR_n)$ if $m \equiv 1, 2, 3 \mod 4$, and $c_m(RLT_n) > c_m(DR_n)$ if $m \equiv 0 \mod 4$. In particular, the last inequality means that DR_n cannot be a maximizer of $c_m(T)$ for each $m \geq 5$.

Adjacency preserving maps

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Tue 11:25, Room B

Two matrices are said to be adjacent if one is a rank one perturbation of the other. The classical Hua's theorems characterize bijective maps on various matrix spaces preserving adjacency in both directions. We will present some recently obtained improvements of these results.

Spectral analysis of inexact constraint preconditioning for saddle point matrices

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Fri 11:50, Room Pacinotti

Linear systems with nonsingular coefficient matrix of the form $\mathcal{A} = [A, B^T; B, 0]$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times n}$, arise in many applications associated with the numerical solution of saddle point problems. We consider the case where $\mathcal{A}$ is symmetric and highly indefinite, so that preconditioning is mandatory if the associated system is to be solved with a Krylov subspace method. In *constrained* optimization problems, preconditioners of the form

$$\mathcal{P} = \begin{bmatrix} I_n & 0 \\ BD^{-1} & I_m \end{bmatrix} \begin{bmatrix} D & 0 \\ 0 & -H \end{bmatrix} \begin{bmatrix} I_n & D^{-1}B^T \\ 0 & I_m \end{bmatrix},$$

where D and H approximate A and $BD^{-1}B^T$, respectively, are particularly effective. In several applications it is sufficient to choose D to be a scaled multiple of the identity, so that efforts focus on the approximation to $BD^{-1}B^T$, which often cannot be computed explicitly [5].

In this talk we present a spectral analysis of the preconditioned matrix $\mathcal{P}^{-1}\mathcal{A}$ as H moves away from its ideal and computationally expensive version $H_{\text{ex}} = BD^{-1}B^T$. Much is known about the spectrum in the ideal case, characterized by a rich spectral structure, with non-trivial Jordan blocks and favourable real eigenvalue distribution [3, 4]. The spectral analysis of the general though far more realistic case $H \neq BD^{-1}B^T$ has received less attention (see, e.g., [1, 2, 5]), possibly due to the difficulty of dealing with Jordan block perturbations. We show that a two-step procedure allows one to successfully handle this complex structure, revealing the true spectral perturbation induced by a workable choice of H.

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[2] H. S. Dollar and A. J. Wathen, *Approximate factorization constraint preconditioners for saddle-point matrices*, SISC, 27(2006), pp. 1555–1572.
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Joint work with V. Simoncini (Università di Bologna)

Jacobi-type algorithms for the Hamiltonian eigenvalue problem

Ivan Slapnicar, Technical University Berlin, Germany
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Mon 17:35, Room Fermi

We present recent results on Jacobi-type algorithms for real Hamiltonian matrices. We describe both, real and complex algorithms. The algorithms use orthogonal (unitary) and non-orthogonal shear transformations. Convergence and accuracy properties of the algorithms are discussed.

Computational aspects of the Moore-Penrose inverse

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Fri 15:00, Room Galilei

In the last years a number of fast algorithms for computing the Moore-Penrose inverse of structured and block matrices have been designed. There is a variety of new papers dealing with numerical algorithms, whose authors neglect the issue of numerical stability of their algorithms and focus only on complexity (number of arithmetic operations). However, very often they are not accurate up to the limitations of data and conditioning of the problem.

In this talk we present a comparison of certain direct and iterative algorithms for computing the Moore-Penrose inverse, from the point of view of numerical stability and algebraic complexity.

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Joint work with Iwona Wróbel (Warsaw University of Technology).

Solving large-scale nonnegative least-squares

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Fri 15:50, Room Galilei

We study the fundamental problem of nonnegative least squares. This problem was apparently introduced by Lawson and Hanson [1] under the name NNLS. As is evident from its name, NNLS seeks least-squares solutions that are also nonnegative. Owing to its wide-applicability numerous algorithms have been derived for NNLS, beginning from the active-set approach of Lawson and Hanson [1] leading up to the sophisticated interior-point method of Bellavia et al. [2]. We present a new algorithm for NNLS that combines projected subgradients with the non-monotonic gradient descent idea of Barzilai and Borwein [3]. Our resulting algorithm is called BBSG, and we guarantee its convergence by exploiting properties of NNLS in conjunction with projected subgradients. BBSG is surprisingly simple and scales well to large problems. We substantiate our claims by empirically evaluating BBSG and comparing it with established convex solvers and specialized NNLS algorithms. The numerical results suggest that BBSG is a practical method for solving large-scale NNLS problems.

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Joint work with D. Kim and I. S. Dhillon (University of Texas at Austin)

The Sinkhorn-Knopp Fixed Point Problem with Patterned Matrices

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Thu 17:10, Room B

We consider the Sinkhorn-Knopp Fixed Point Problem

$$(A^T(A\vec{x})^{(-1)})^{(-1)} = \vec{x}$$

where (-1) is the entry-wise inverse of a vector. This problem arises from work originally done in the 1960s by Sinkhorn and Knopp [1] for transforming a matrix into a doubly stochastic matrix by the pre- and post-multiplication of a matrix by diagonal matrices: D_1AD_2. This process has a variety of applications, including most recently in web page rankings, e.g. in a Google search. We have investigated the types of solutions that arise in solving this fixed point equation both in the general case and for specific types of matrices, in particular, patterned matrices. The results in the circulant case are particularly interesting and exhibit a very cyclical behavior. We share results for the circulant case and other cases involving various other types of patterned matrices, and relate our results to the original problem of trying to transform a matrix into a doubly stochastic one. The majority of this work was done by undergraduates under my supervision.

[1] Richard Sinkhorn and Paul Knopp, *Concerning nonnegative matrices and doubly stochastic matrices*, Pacific J. Mathematics **21** (1967), pp. 343 - 348.

Spectraloid operator polynomials, the approximate numerical range and an Eneström-Kakeya theorem in Hilbert space

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Fri 11:00, Room Galilei

We study a class of operator polynomials in Hilbert space, which are spectraloid in the sense that spectral radius and numerical radius coincide. The focus is on the spectrum in the boundary of the numerical range. As an application the Eneström–Kakeya–Hurwitz theorem on zeros of real polynomials is generalized to Hilbert space.

Joint work with H. K. Wimmer (Universität Würzburg, Germany)

Bifurcation analysis of eigenvalues of polynomial matrices smoothly depending on parameters

S. Tarragona, Universidad de León, Spain
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Thu 11:25, Room A

Let $P(\lambda) = \sum_{i=0}^{k} \lambda^i A_i(p)$ be a family of monic polynomial matrices smoothly dependent on a vector of real parameters $p = (p_1, \ldots, p_n)$. In this work we study behavior of an eigenvalue of the monic polynomial family $P(\lambda)$.

[1] A. P. Seyranian, A.A. Mailybaev, "Multiparameter Stability Theory with Mechanical Applications" World Scientific, Singapore, 2003.
[2] Mª I. García, *Introducción a la Teoría de Matrices Polinomiales*. Edicions UPC, Barcelona, 1999.
[3] I. Gohberg, P. Lancaster, L. Rodman, "Matrix Polynomials", Academic Press, New York, 1982.
[4] G.W. Stewart, J. Sun, "Matrix Perturbation Theory", Academic Press, New York, 1990.

Joint work with M. I. García-Planas (Universitat Politècnica de Catalunya)

A class of matrices generalizing the idempotent ones

N. Thome, Universidad Politécnica de Valencia, Spain
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Tue 11:50, Room B

In the last years, the concept of idempotency, tripotency and, in general, $\{k+1\}$-potency has been studied from a different point of view in the literature. For example, the case when linear combination of two idempotent (tripotent, $\{k+1\}$-idempotent) matrices is idempotent (tripotent, $\{k+1\}$-idempotent) has been developed [1,2,3]. It seems to be natural to extend the idea of $\{k+1\}$-potency. In this work, we present such a generalization and we study this kind of matrices giving some properties of them.

This paper has been partially supported by DGI grant MTM2007-64477 and by grant Universidad Politécnica de Valencia, PAID-06-09, Ref.: 2659.

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[2] J.K. Baksalary, O.M. Baksalary, G.P.H. Styan. Idempotency of linear combinations of an idempotent matrix and a tripotent matrix. Linear Algebra and its Applications 354, 21-34, 2002.
[3] J. Benítez, N. Thome. $\{k\}$-group periodic matrices. SIAM J. Matrix Anal. Appl. 28, 1, 9-25, 2006.

Joint work with L. Lebtahi (Universidad Politécnica de Valencia)

On efficient numerical approximation of the scattering amplitude

P. Tichý, Czech Academy of Sciences, Czech Republic
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Tue 15:50, Room B

This talk presents results on efficient and numerically well-behaved estimation of the scalar value c^*x, where c^* denotes the conjugate transpose of c and x solves the linear system $Ax = b$, $A \in \mathbb{C}^{N\times N}$ is a nonsingular complex matrix and b and c are complex vectors of length N. In other words, we wish to estimate the *scattering amplitude* $c^*A^{-1}b$.

In our understanding, various approaches for numerical approximation of the scattering amplitude can be viewed as applications of the general mathematical concept of *matching moments model reduction*, formulated and used in applied mathematics by Vorobyev in his remarkable book [3]. Using the Vorobyev moment problem, matching moments properties of Krylov subspace methods can be described in a very natural and straightforward way, see [1]. This talk further develops the ideas from [1] into efficient estimates of $c^*A^{-1}b$, see [2].

We briefly outline the matching moment property of the Lanczos and Arnoldi algorithms, and specify techniques for estimating $c^*A^{-1}b$ with A non-Hermitian, including a new algorithm based on the BiCG method. We show its mathematical equivalence to the existing estimates which use a complex generalization of Gauss quadrature, and discuss its numerical properties. The proposed estimate will be compared with existing approaches using analytic arguments and numerical experiments on a practically important problem that arises from the computation of diffraction of light on media with periodic structure.

[1] Z. Strakoš, Model reduction using the Vorobyev moment problem, Numer. Algor., Vol. 51, pp. 363–379, July, 2009.
[2] Z. Strakoš and P. Tichý, On efficient numerical approximation of the scattering amplitude $c^*A^{-1}b$ via matching moments, submitted, 2009.
[3] Y. V. Vorobyev, Methods of moments in applied mathematics, Gordon and Breach Science Publishers, New York, 1965.

Joint work with Z. Strakoš (Czech Academy of Sciences, Czech Republic)

Computation of the greatest common divisor of polynomials through Sylvester matrices and applications in image deblurring

D. Triantafyllou, University of Athens, Greece
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Fri 15:50, Room B

We present new, fast methods computing the greatest common divisor (GCD) of polynomials. We develop algorithms computing in an efficient way the upper triangularization of the Modified Sylvester (MS) [4] and Modified Generalized Sylvester (MGS) matrix, resulting to the GCD of polynomials. All methods are exploiting the special structure of MS and MGS matrices, reducing by one order the required complexity in case of several polynomials. For this case, we propose also a fast parallel method for the computation of their GCD using

Housholders' transformations, improving significant the complexity of the classical procedure. The use of floating point arithmetic can lead to incorrect GCDs. We study the behavior of the numerical implementation of the methods in respect of the inner accuracy ϵ_t of the procedures and the significant digits that are used. Various values of these quantities many times can lead to different GCDs or comprimeness [2]. The complexity is computed for all methods and tables comparing them with the known techniques [1,3] are given. All the algorithms are tested for several sets of polynomials and the results are summarized in tables, resulting to useful conclusions. Finally, an interesting application in image deblurring is given.

[1] S. Barnett, Greatest common divisor of several polynomials, Proc. Cambridge Phil. Soc., 70, pp. 263-268, 1971.
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[3] I. S. Pace and S. Barnett, Comparison of algorithms for calculation of g.c.d. of polynomials, Internat. J. System Sci., 4, pp. 211-226, 1973.
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Joint work with M. Mitrouli (University of Athens)

A preconditioning approach to the Google pagerank computing problem

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Wed 12:40, Room A

It is well known that the Google pagerank vector p can be computed by solving a sparse linear system $Ax = b$. In this talk we first show that such system can be solved by the Euler-Richardson (ER) method with the same computational complexity of the power method (which is the standard algorithm for computing p). Then we observe that, by the particular structure of the Google matrix, only one eigenvalue of A is responsible of the low rate of convergence of ER, and that such eigenvalue can be removed by preconditioning A. In fact, applying ER to $\mathcal{A}^{-1}Ax = \mathcal{A}^{-1}b$ for a suitable choice of the preconditioner $\mathcal{A}$ improves the convergence rate roughly from 0.85^k to 0.25^k. Further studies to reduce the surplus of operations per step are in progress.

Joint work with Carmine Di Fiore (difiore@mat.uniroma2.it)

Extreme Distance Field of Values Points: How to Compute?

Frank Uhlig, Auburn University, Auburn, AL, USA
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Tue 15:25, Room B

The field of values $F(A) = \{x{*}Ax \in \mathbb{C} | x \in \mathbb{C}^n\}$ of a square $n\times n$ matrix A contains highly useful information of A. How can the extreme distance points of $F(A)$ from zero be computed for a given A? Why are these distances important?

In particular, why: the maximal distance, called the numerical radius $r(A)$, of points p in $F(A)$ from zero determines the transient behavior of the system governed by A while a positive minimal distance between zero and $F(A)$, called the Crawford number, insures stability of the system. These related problems have quite different flavors: the spectral radius can be achieved at multiple and even infinitely many points on the boundary of $F(A)$ regardless of where zero lies, but if zero lies outside the field of values then the Crawford number is realized at only one point on the boundary of $F(A)$. And if zero lies inside $F(A)$, then the (negative) generalized Crawford number can occur at any number of $F(A)$ boundary points.

We explain and compare new fast and accurate vector and geometry based algorithms with recent, but much slower optimization type algorithms for these two problems.

Lyapunov Equation Methods for Solving the Matrix Nonlinear Schrödinger Equation

Cornelis van der Mee, Dip. Matematica e Informatica, Università di Cagliari
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Thu 15:25, Room C

We derive exact $n\times m$ matrix solutions of the focusing matrix nonlinear Schrödinger (NLS) equation

$$iu_t + u_{xx} + 2uu^\dagger u = 0, \qquad (x,t) \in \mathbb{R}^2, \tag{1}$$

by considering $u(x,t)$ as the potential in the matrix Zakharov-Shabat system

$$\frac{\partial \Psi}{\partial x}(x,\lambda) = \begin{pmatrix} i\lambda I_n & -u(x,t) \\ u(x,t)^\dagger & -i\lambda I_m \end{pmatrix} \Psi(x,\lambda), \qquad x \in \mathbb{R}, \tag{2}$$

and applying inverse scattering of Eq. (2) by solving the coupled Marchenko integral equations. Exploiting the Hankel structure of its integral kernels and representing them in the form

$$\Omega(x+y) = Ce^{-(x+y)A}e^{4itA^2}B \tag{3}$$

for suitable matrix triplets (A,B,C), exact solutions $u(x,t)$ of Eq. (1) are obtained, requiring a careful analysis of two Lyapunov matrix equations. Next, we discuss various transformations to generate matrix NLS solutions from other such solutions.

Joint work with Francesco Demontis (Università di Cagliari)

Matrix algebras can be spectrally equivalent with ill conditioned Toeplitz matrices

P. Vassalos, Athens University of Economics and Business, Greece
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Fri 11:25, Room Pacinotti

In this work, we prove the existence of matrices, $\tau_n(f)$, belonging to τ algebra that are spectrally equivalent with ill conditioned Toeplitz matrices $T_n(f)$. For that, we assume that the generating function f is real valued, nonnegative, continuous, with isolated roots of maximum order $\alpha \in \mathbb{R}^+$. Specifically, we prove that for $0 \le \alpha \le 2$ there exist a proper clustering of the eigenvalues of $\tau_n(f)^{-1}T_n(f)$ around unity. For $2 < \alpha < 4$, a weak clustering for the spectrum of the aforemention matrix is achieved, where the minimum eigenvalue is bounded from bellow, while a constant number, independent of n, of eigenvalues tend to infinity. The results are generalized to cover the more interesting, from theoretical and practical point of view, case of Block Toeplitz with Toeplitz Blocks (BTTB), matrices. Based on these theoretical statements we propose τ preconditioners that lead to superlinear convergence both in $1D$ and $2D$ case when the condition number of the Toeplitz matrix is $o(n^4)$. Finally, we show that the spectrally equivalence also holds between circulant matrices and ill-conditioned

Toeplitz matrices. The main difference is that the continuous symbol which generates the Toeplitz matrix should have discrete roots of order less than 2. We perform many numerical experiments, whose results confirm the validity of theoretical analysis.

Joint work with D. Noutsos (University of Ioannina)

Francis's Algorithm

David S. Watkins, Washington State University
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Mon 16:45, Room Fermi

Francis's implicitly-shifted QR algorithm has for many years been the most widely used algorithm for computing eigenvalues of matrices. The standard (and time-honored) method of justifying Francis's algorithm is to show that each iteration is equivalent to a step (or several steps) of the explicitly-shifted QR algorithm. In this talk we will argue that the standard approach is unduly complicated. Instead one should argue directly that Francis's algorithm performs nested subspace iterations with a change of coordinate system at each step. This is done by bringing to light the role of the nested Krylov subspaces that lurk in the transforming matrices.

Block diagonalization for a matrix A when $AR = RA$ and $R^k = I$

James R. Weaver, Dept. of Mathematics/Statistics, University of West Florida, Pensacola, USA
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Thu 15:00, Room Galilei

This article examines the block diagonalization of a $n \times n$ complex matrix A when $AR = RA$ for a general $n \times n$ complex matrix $R$with the property that $R^k = I$ for a positive integer k. First find the Jordan Form for the matrix R, which is a diagonal matrix D in the case $R^k = I$, and a corresponding transforming matrix P. This information is used to find a block diagonalization of A given some additional information about the matrix A.

Commutators with maximal Frobenius norm

D. Wenzel, Chemnitz University of Technology, Germany
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Fri 16:45, Auditorium

When investigating the commutator of two normed $n \times n$ matrices, two situations are of special interest: the commuting case (i.e. the commutator is zero) and the "maximal non-commuting case" (in which the commutator has a norm as large as possible). Regarding matrices at random typically yields pairs very close to commutativity – especially if their size n is large. Although actually none of these matrices really commute, except for 2×2 matrices it is seemingly hopeless to find a pair whose commutator admits Frobenius norm $\sqrt{2}$, which is the known bound for the maximal situation.

We will present an explanation for that behaviour by determining all pairs of real or complex matrices satisfying the equality

$$\|XY - YX\|_{\mathsf{F}} = \sqrt{2}\|X\|_{\mathsf{F}}\|Y\|_{\mathsf{F}}.$$

The result is a surprisingly simple and meager, but also nicely structured set. The talk is based on joint work with Che-Man Cheng and Seak-Weng Vong.

[1] C.-M. Cheng, S.-W. Vong, D. Wenzel, Commutators with maximal Frobenius norm, Linear Algebra Appl. 432 (2010) 292–306.

Joint work with Che-Man Cheng and Seak-Weng Vong (University of Macau)

Hyperinvariant, characteristic and marked subspaces

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Mon 11:25, Room B

Abstract: Let V be a finite dimensional vector space over a field K and f a K-endomorphism of V. We study three types of f-invariant subspaces, namely hyperinvariant subspaces, which are invariant under all endomorphisms of V that commute with f, characteristic subspaces, which remain fixed under all automorphisms of V that commute with f, and marked subspaces, which have a Jordan basis (with respect to $f_{|X}$) that can be extended to a Jordan basis of V. We show that a subspace is hyperinvariant if and only if it is characteristic and marked. If K has more than two elements then each characteristic subspace is hyperinvariant.

Joint work with Pudji Astuti (ITB Bandung)

Structured matrix methods for the computation of multiple roots of inexact polynomials

Joab Winkler, University of Sheffield, United Kingdom
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Fri 15:00, Room B

This paper considers the application of structured matrix methods for the calculation of a structured low rank approximation of the Sylvester resultant matrix of two polynomials that are corrupted by additive noise. It is shown that this low rank approximation allows the computation of multiple roots of inexact polynomials that have been corrupted by additive noise, such that the multiplicities of the theoretically exact roots are preserved. Particular problems occur with polynomials whose coefficients vary widely in magnitude, and it is shown that these polynomials must be processed prior to the computation of their roots.

The talk will contain computational results that demonstrate the theoretical analysis.

Joint work with Madina Hasan (University of Sheffield) and Xin Lao (University of Sheffield)

On the numerical range of companion matrices

Iwona Wróbel, Warsaw University of Technology, Poland
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Fri 11:25, Room Galilei

It is known that the convex hull of the roots of a given polynomial contains the roots of its derivative. This result is known as the Gauss-Lucas theorem. We will investigate the possibility of generalizing it to the numerical range of companion matrices and discuss the relation between the numerical ranges of companion matrices of a polynomial and its derivative. Several types of companion matrices will be considered.

Matrix inequalities associated with the data processing inequality

Masahiro Yanagida, Tokyo University of Science, Japan
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Mon 16:45, Room C

In the cascade of two channels $X \to Y \to Z$, a refined version of the data processing inequality of the form $\frac{I(X,Z)}{I(X,Y)} \le c(A)$ has been found by Evans and Schulman [1] for binary channels, where the bound $c(A)$ (≤ 1 generally) depends only on the channel matrix A of the second channel $Y \to Z$. In this report we give a general observation that may help us to find such bounds for non-binary channels, and find one for a certain symmetric A.

[1] W. S. Evans and L. J. Schulman, *Signal propagation and noisy circuits*, IEEE Trans. Inform. Theory **45** (1999), no. 7, 2367–2373.