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## Linear Algebra Education

Avi Berman, Technion - Israel Institute of Technology,  
Haifa, Israel  
Steve J. Leon, University of Massachusetts, USA

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### Principles and tools in teaching linear algebra

AVI BERMAN, Tachnion, Haifa, Israel

*Fri 12:15, Room A*

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### What Have I Learned?

JANE DAY, Mathematics Department San Jose State University

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*Fri 11:00, Room A*

Linear algebra is my favorite subject to teach. I've always been interested in how people learn and in teaching styles that might help students better understand and appreciate the power of linear algebra. I've been impressed by insights from the Linear Algebra Curriculum Study Group, the MAA, Jean Piaget, Maria Montessori, and from Guershon Harel and other colleagues. I've learned from observing other people's classes. I've seen that students learn differently, that some but not all really benefit from geometric visualization, applications, computer use and/or group work, and that my (supposedly crystal clear) phrasing can be ambiguous to them. I've found that student attitudes can be very different at another college. I will discuss such experiences and some methods I've tried that seemed to help or not.

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### To be announced

GUERSHON HAREL, University of California, San Diego, USA

*Tue 16:45, Room A*

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### The use of Classroom Response Systems (clickers) in teaching linear algebra: Still more questions than answers

BORIS KOICHU, Technion Israel Institute of Technology

*Tue 17:10, Room A*

Classroom Response Systems (clickers) are available and rapidly disseminating technology of enhancing interactions in large-size classes. This technology enables students to individually respond to various multiple-choice questions asked by the lecturer during the lesson. The talk will focus on the following questions: What are some of the strategies of using clickers in large-size linear algebra classes? What are the affordances and limitations? What are the effects of using clickers on the students' learning, teaching practices and pedagogical knowledge of the lecturers? These questions will be discussed based on the review of the growing literature on the subject and on the preliminary results of an on-going teaching experiment conducted in the context of two basic linear algebra courses at the Technion.

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Joint work with Eman Atrash, Israel Institute of Technology

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### Contents of Linear Algebra with Sage and Mobile Sage environment

SAN-GU LEE, Sungkyunkwan University, Korea

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*Fri 11:50, Room A*

From the experience gained by students in our Linear Algebra class, we wish that our students can attain a better understanding of mathematical concepts and further they can be equipped with a tool to deal with some real world problems. Sage is an open-source mathematics software system. It combines the power of many existing open-source packages into a common Python-based interface. We have tried to adopt Sage for our Linear Algebra class. It worked beautifully without much cost. We will try to share our contents and experience that came from teaching of Linear Algebra in Sage and Mobile Sage environment.

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Joint work with Duk-Sun Kim

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### The second undergraduate level course in linear algebra

STEVE LEON, University of Massachusetts Dartmouth, USA

*Fri 15:00, Room A*

In this talk we will review briefly the recommendations made twenty years ago by the NSF sponsored Linear Algebra Curriculum Study Group. We will discuss what topics should be covered in undergraduate linear algebra courses and give reasons why we believe a second course in linear algebra should be required for all mathematics majors. The speaker will outline a number of alternatives for possible second courses. He will describe one such course where students work together in teams on projects and apply linear algebra to problems in areas such as digital imaging, computer animation, and coordinate metrology. Some of these projects may involve original undergraduate level research.

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### Using an economics model for teaching linear algebra

EDGARD POSSANI, Instituto Tecnológico Autónomo de México, México

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*Tue 17:35, Room A*

In this talk we will present an approach to teaching linear algebra using models. In particular, we are interested in designing problems that meet the models and modeling [1] approach, and in analyzing students learning process under an APOS [2] perspective. We will present a short illustration of the analysis of an economics problem related to production. This problem elicits the use of several linear algebra concepts related to vector space. Previous work has highlighted the importance of using realistic problems in the teaching of linear algebra. Here we will address the use of learning trajectories, which together with a genetic decomposition has allowed us to design specific teaching sequences to help students develop the constructions needed to learn the desired concepts. The abovementioned economics problem has already been used in the classroom by several researchers who also teach linear algebra. We will present an analysis of the learning trajectory, and describe the actual learning process, its outcomes together with students' difficulties and modeling strategies. In the process of solving the problem students need to analyze specific sets of data, and this analysis helps them discriminate which data sets are better suited for finding unique solutions for the problem, and which conditions are necessary for the selection of the appropriate data. We have found that this problem helps the students give meaning to more complex algebra concepts such as base, linear independence, generating set,

among others related to vector space. The realistic setting of the problem motivates them to carry out a deeper kind of mathematical analysis. We believe this approach promotes students' significant development of mathematical reasoning in a meaningful and realistic setting.

[1] R. Lesh & H. Doerr. *Beyond Constructivist: A Model & Modelling Perspective on Mathematics Teaching, Learning and Problems Solving*, Laurence Erlbaum Associates NH, 2003.

[2] E. Dubinsky. *Reective Abstraction in Advanced Mathematical Thinking*, in *Advanced Mathematical Thinking* (D. Tall, ed.), Kluwer (1991), 95-126.

Joint work with M. Trigueros (ITAM), G. Preciado (ITAM), D. Lozano (ITAM)

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### **Questions about Teaching, Teaching Mathematics and Teaching Linear Algebra**

FRANK UHLIG,

*Fri 11:25, Room A*

Questions are the engine of our understanding and food for the development of our consciousness. What is there? What should be there? What am I doing or trying to do? Why so? What are we doing? How can I achieve my goal(s)? How will we achieve our goals? What are the intended consequences, the unintended ones? Can my goal(s) be achieved? What are the costs and success? Can our goals be achieved, at what cost and success? Questions of What, Why, How? and their negations What not, Why not, How not? all play a helpful role in assessing our individual and group efforts and the success of teaching the next and after next generation.

Question 1: Why are we still teaching math today? (Why not just calculator/computer literacy?)

Question 2: Why are we teaching linear algebra and matrix theory? (Why not just introduce MATLAB in freshman year?)

Question 3: Why are we teaching known unstable algorithms in college algebra and linear algebra courses? Why are our textbooks full of these? (Why not give our students correct and useful information?)

Question 4: Why is linear (in)dependence the students' and our stumbling block in linear algebra courses and books? (Why not progress to eigen structures of normal matrices, the Schur normal form, matrix factorizations and the SVD?)

Question 5: Why are US high school graduates lagging behind? (Why not teach concepts and exploration from Kindergarten on?)

Question 6: Who am I? For and by myself, and for my students?

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