ESERCIZIO 22

$$\lim_{x\to+\infty} \frac{3x^3+x+1}{x^3+2} = \lim_{x\to+\infty} \frac{\cancel{x^3}\left(3+\frac{1}{x^2}+\frac{1}{x^3}\right)}{\cancel{x^3}\left(1+\frac{2}{x^3}\right)} = \frac{3}{1} = 3$$

$$\lim_{x\to+\infty} \underset{\downarrow\,+\infty}{3x^3} + \underset{\downarrow\,+\infty}{x} + 1 = +\infty \qquad \lim_{x\to+\infty} \underset{\downarrow\,3}{3} + \underset{\downarrow\,0}{\frac{1}{x^2}} + \underset{\downarrow\,0}{\frac{1}{x^3}} = 3+0+0 = 3$$

$$\lim_{x\to+\infty} x^3+2 = +\infty \qquad \lim \underset{\downarrow\,1}{1} + \underset{\downarrow\,0}{\frac{2}{x^3}} = 1$$

$$\lim_{x\to-\infty} \frac{3x^3+x+1}{x^2+2} = \lim_{x\to-\infty} \frac{\cancel{x^3}\left(3+\frac{1}{x^2}+\frac{1}{x^3}\right)}{\cancel{x^2}\left(1+\frac{2}{x^2}\right)} =$$

$$\lim_{x\to-\infty} \underset{\downarrow\,-\infty}{3x^3} + \underset{\downarrow\,-\infty}{x} + \underset{\downarrow\,1}{1} = -\infty$$

$$\lim_{x\to-\infty} \underset{\downarrow\,+\infty}{x^2} + \underset{\downarrow\,2}{2} = +\infty$$

$$= \lim_{x\to-\infty} \underbrace{x}_{\to -\infty} \boxed{\overbrace{\frac{3+1/x^2+1/x^3}{1+2/x^2}}^{\to 3}} = -\infty$$

$$\lim_{x\to-\infty} \underset{\downarrow\,3}{3} + \underset{\downarrow\,0}{\frac{1}{x^2}} + \underset{\downarrow\,0}{\frac{1}{x^3}} = 3$$

QUINDI $\lim_{x\to-\infty} \frac{3+1/x^2+1/x^3}{1+2/x^2} = 3$

$$\lim_{x\to-\infty} 1+\frac{2}{x^2} = 1.$$

ESERCIZIO 23

$(e^3)^x$

$$\lim_{x\to+\infty} \frac{3e^{3x}+e^x+1}{e^3x+2} = \lim_{x\to+\infty} \underbrace{\frac{e^{3x}}{x}}_{\to +\infty} \; \underbrace{\frac{3+e^{-2x}+e^{-3x}}{e^3+2x^{-1}}}_{\to \frac{3}{e^3}}$$

Numeratore e denominatore tendono a $+\infty$

$= +\infty.$

$$\lim_{x\to+\infty} \underset{\substack{\downarrow\\3}}{3} + \underset{\substack{\downarrow\\0}}{e^{-2x}} + \underset{\substack{\downarrow\\0}}{e^{-3x}} = 3$$

$$\lim_{x\to+\infty} \underset{\substack{\downarrow\\e^3}}{e^3} + \underset{\substack{\downarrow\\0}}{2x^{-1}} = e^3$$

$$\lim_{x\to+\infty} \frac{\overbrace{3e^{3x}+e^{x}+1}^{\to+\infty}}{\underbrace{e^{3x}+2}_{\to+\infty}} = \lim_{x\to+\infty} \frac{\cancel{e^{3x}}}{\cancel{e^{3x}}} \frac{3+\overbrace{e^{-2x}}^{\to 0}+\overbrace{e^{-3x}}^{\to 0}}{1+2\underbrace{e^{-3x}}_{\to 0}} = \frac{3}{1}$$

DALL'ESR. PRECEDENTE.

$$\lim_{x\to+\infty} \frac{3e^{3x}+e^{x}+1}{e^{3x}+2} = f(x) = \lim_{y\to+\infty} \frac{3y^3+y+1}{y^3+2} = \frac{3}{1} = 3.$$

$$x \longrightarrow e^{x} = y \longrightarrow \frac{3y^3+y+1}{y^3+2}$$

$y^3 = (e^x)^3 = e^{3x}$

$$x \longrightarrow e^{5} = y$$

Se x tende a $+\infty$ y tende a $+\infty$

ESEMPIO

$$\lim_{x\to 0} \frac{e^{x}-1}{x}$$

$$\lim_{x\to 0} e^{x}-1 = e^{0}-1 = 0$$

$$\lim_{x\to 0} x = 0$$

$$x \longrightarrow y = e^{x}-1 \longrightarrow \frac{e^{x}-1}{x} = \frac{y}{x} = \frac{y}{\log(1+y)}$$

$$y+1 = e^{x}$$

$$\log_e(y+1) = x$$

Quando x tende a 0 y tende a 0.

$$\lim_{x\to 0} \frac{e^x-1}{x} = \lim_{y\to 0} \frac{y}{\log(y+1)} = \lim_{y\to 0} \frac{1}{\frac{1}{y}\log(1+y)} = 1$$

$1 \to 1$, $\frac{1}{y}\log(1+y) \to 1$

$$\left(\log(a^b) = \alpha \qquad e^{\alpha} = a^b \right.$$

$$\log a = \beta \qquad e^{\beta} = a$$

$$e^{b\beta} = (e^{\beta})^b = a^b$$

QUINDI $\alpha = b\beta$

$$\left. \underbrace{\log(a^b) = b \log a} \right)$$

$$\boxed{\lim_{y\to 0} \underline{\frac{1}{y}\log(1+y)}} = \lim_{y\to 0} \log\left[\boxed{(1+y)^{1/y}}\right] = \log e = 1$$

$(1+y)^{1/y} \to e$

$$e = \lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n = \lim_{y\to 0} (1+y)^{1/y}$$

$1/n = y$ $\qquad$ $n \to +\infty$, $y \to 0$

$$\lim_{x\to 0} f\left(\underbrace{e^x+1}_{=y}\right) = \lim_{y\to 2} f(y)$$

$e^x+1 \to 2$

dove $y = e^x + 1$

$$f(y) = \sin y + \cos y + \operatorname{arctg} y + \frac{1}{y^2+1}$$

$$= \sin(1+e^x) + \ldots \ldots$$

ESERCIZIO 27 $\qquad f(x) = \log_2(x^2+1)$

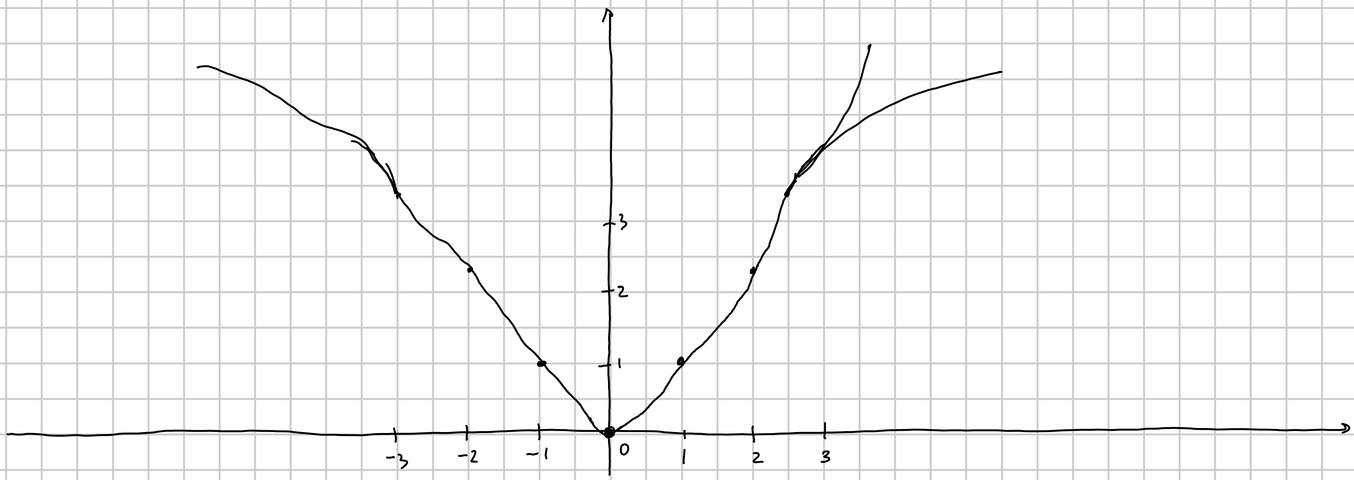
0

$x = 0 \quad \log_2(1) = 0 \qquad x = 1 \quad \log_2(1+1) = \log_2 2 = 1$

$x = 2 \qquad \log_2(1+4) = \log_2 5 = 2{,}32$

$x = 3 \qquad \log_2(1+9) = \log_2 10 = 3{,}32$

x	-3	-2	-1	0	1	2	3
$f(x)$	3,32	2,32	1	0	1	2,32	3,32



$$\lim_{x \to +\infty} \log_2(\underbrace{1+x^2}_{\Downarrow \; y \to +\infty}) = \lim_{y \to +\infty} \log_2(y) = +\infty$$

$$\lim_{x \to -\infty} \log_2(\underbrace{1+x^2}_{= y}) = \lim_{y \to +\infty} \log_2(y) = +\infty$$

se x tende a $-\infty$, y tende a $+\infty$.

$x = 0$ è un punto di minimo

e quindi $\log_2(1+0^2) = 0$ è il valore minimo.

voglio far vedere che $\log_2(1+x^2) \geq 0$ per ogni x.

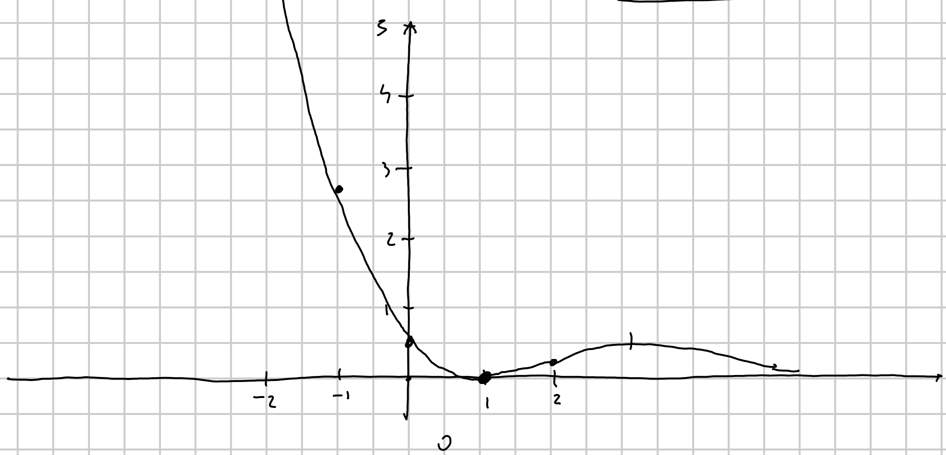
Se $x \in \mathbb{R}$ $\quad 1 + x^2 \geq 1$ e quindi

$$\log_2(1+x^2) \geq \log_2 1 = 0.$$

ESERCIZIO 29

$$f(x) = \frac{(x-1)^2}{1+2^x}$$

$$\boxed{\lim_{x \to -\infty} f(x) = +\infty} \qquad \boxed{\lim_{x \to +\infty} f(x) = 0}$$



x	$f(x)$
-2	$7,2$
-1	$2,\overline{6}$
0	$0,5$
1	0
2	$0,2$

LA FUNZIONE NON PUÒ AVERE MASSIMO PERCHÉ $\lim_{x \to -\infty} f(x) = +\infty$ E QUINDI ASSUME VALORI GRANDI A PIACERE.

OSSERVAZIONE $\quad f(x) = \dfrac{\overbrace{(x-1)^2}}{\underbrace{1+2^x}} \geq 0$

PERCHÉ È IL RAPPORTO DI DUE NUMERI MAGGIORI O UGUALI A ZERO. INOLTRE $0 = f(1)$ E QUINDI 0 È IL VALORE MINIMO E 1 È UN PUNTO DI MINIMO.

ESERCIZIO 30

$f: \mathbb{R} \to \mathbb{R}$

A. $f(x) > 0$ per ogni x

B. f è crescente

C. $f(0) = 1$

D. $\lim_{x \to +\infty} \frac{1}{x} f(x) = 1$

1ª PROPOSTA $\quad f(x) = e^x \quad$ A ✓ $\quad$ B ✓ $\quad$ C ✓ $\quad \lim_{x \to +\infty} \frac{e^x}{x} = +\infty$

D NON VA BENE

3ª PROPOSTA $\quad f(x) = \dfrac{x^3+1}{x^2+1}$ NON È SEMPRE POSITIVA

PER ESEMPIO $\quad f(-2) = \dfrac{-8+1}{5} = -\dfrac{7}{5} < 0.$

2ª PROPOSTA $\quad f(x) = \log_2(1+e^x)$

(A) È SEMPRE POSITIVA OK

$1 + e^x > 1$ E QUINDI $\log_2(1+e^x) > \log_2(1) = 0.$

(B) È CRESCENTE SE $x_1 < x_2$

ALLORA $e^{x_1} < e^{x_2}$ QUINDI $1+e^{x_1} < 1+e^{x_2}$

E INFINE $\log_2(1+e^{x_1}) < \log_2(1+e^{x_2})$

(C) $\log_2(1+e^0) = \log_2(2) = 1.$

(D) $$\lim_{x\to+\infty} \frac{\log_2(1+e^x)}{x} =$$

$\log_B a^b = b\log_B a$

$$\lim_{x\to+\infty} \frac{\log_2(1+e^x) - \log_2(e^x) + \log_2(e^x)}{x} =$$

$$= \lim_{x\to+\infty} \frac{\log_2\left(\frac{1+e^x}{e^x}\right)}{x} + \boxed{\frac{\log_2 e^x}{x}}$$

$\log_B a - \log_B b = \log_B\left(\frac{a}{b}\right)$

0 $\quad \log_2 e$

$\log_2\left(\frac{1+e^x}{e^x}\right) = \log_2\left(\frac{1}{e^x}+1\right) \to 0$ $\quad \frac{1}{e^x} \to 0$

quindi $\lim_{x\to+\infty} f(x) = \log_2 e \neq 1.$

$\log_2 e \;\text{"}\; 1, \ldots$

ESERCIZIO 31

$f : \mathbb{R} \to \mathbb{R}$

1) $f(x+1) = f(x)$ per ogni x

2) $f(1) = 0.$

$$\boxed{\begin{aligned} &g(x+2\pi) = g(x) \\ &g(0) = 0 \\ &\sin(x+2\pi) = \sin(x) \\ &\sin(0) = 0. \end{aligned}}$$

$f(x) = \boxed{\sin(2\pi x)}$

$$\begin{aligned} f(x+1) &= \sin(2\pi(x+1)) = \\ &= \sin(2\pi x + 2\pi) = \\ &= \sin(2\pi x) = f(x) \end{aligned}$$

$f(1) = \sin(2\pi \cdot 1) = \sin(2\pi) = \sin(0) = 0$

$f(x) = \boxed{\cos(2\pi x)}$

$$\begin{aligned} f(x+1) &= \cos(2\pi x + 2\pi) \\ &= \cos(2\pi x) \\ &= f(x) \end{aligned}$$

$f(1) = \cos(2\pi) = 1.$

$\boxed{\sin(2\pi x)}$ VA BENE