TUTORIAL: MATRIX ANALYTIC METHODS FOR FLUID FLOW MODELS

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Stochastic fluid flow models have been used extensively in a variety of areas such as dam and storage theories, insurance risk, and performance modeling. The canonical model of this type assumes a random environment governed by a finite state Markov chain which modulates the linear rate of change of the fluid level such that when the Markov chain is in state i, the fluid changes at rate c_i per unit time.

Noting the model's similarity to a QBD, in 1999 I developed a steady state analysis of fluid flow models pursuing ideas similar to those in matrix geoemtric methods for Quasi-birth-and-death processes. Subsequently this theory has been developed further by me in several papers with Soohan Ahn. Overall, the approach has proven to be quite powerful in a variety of ways: (a) it reduces the continuous time, continuous state space problem to the consideration of a discrete state space QBD in discrete time; (b) it provides powerful algorithms that do not appear to suffer from the numerical instabilities of other types of methods; (c) it provides a systematic framework to analyze fluid flows and is based on probabilistic methods and path properties. The method is particularly interesting as it offers a potential for significant generalization and thereby opens up several interesting areas for further research.

The goal of this tutorial is to review the basic ideas and methods behind the matrix-geometric approach to fluid flow models. While key results can be derived by elementary methods, a rigorous approach justifying the steps necessitates some advanced tools like stochastic discretization, stochastic coupling, and stochastic process limits; we shall therefore also provide an intuitive understanding of this interplay with probability enabling an easier reading of the set of our papers in this area.