NONLINEAR GENERALIZED FUNCTIONS AND INFINITESIMALS

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25 years ago I found a nonlinear theory of generalized functions in a purely standard style (I ignored mathematical logics and non-standard analysis): [J. Math. Ana. Appl. 94, 1983, pp. 96-115; the book *New Generalized Functions*, North-Holland 1984, chap. 3]. This theory involves infinitesimal generalized functions (and numbers) that follow immediately from the definition of generalized functions.

Despite appearances, the following (sample of) applications (immediately accessible at the intuitive level) will be shown to be closely related: *a miracle of infinitesimals*; no prerequisites are needed:

pure mathematics (1981). A general multiplication of distributions: the Schwartz proof of non-existence of any nonlinear theory of generalized functions relies on the implicit refusal of infinitesimals.

engineering and applied mathematics (1984-89). Efficient numerical schemes to design armour of battle tanks: meaningless products of distributions in equations of physics can be mastered by a suitable use of infinitesimals in a deeper statement of the laws of physics, yielding new formulas to be checked experimentally, then of immediate use to engineers through Godunov type numerical schemes.

mathematical physics (if not allowed by time, see the original works: Roland Steinbauer, Michael Kunzinger, J.Math.Phys. and Class.Quant.Grav. 1997-2000, arxiv.). Explanation of a paradox (the Reissner-Nordstrom field of a very fast rotating black hole) and mathematical understanding of an intuitive method of physics (the paste-scissors method of Penrose for impulsive gravitational waves).

A great number of applications to numerous branches of mathematics and to mathematical physics and engineering, about 400 papers or more, have been published (see Math Reviews, Zbltt, Arxiv, ...). Since they rely on infinitesimals (through nonlinear generalized functions) I suggested a proposal of collaboration to the non-standard analysis community [Springer Lecture Notes in Math 1532, 1992, p5] that might still be of interest.[1]

References: a few research-expository texts: Colombeau, Bull. AMS, 23, 2, 1990, pp. 251-268; Egorov, Russian Math. Surveys 45,5, 1990, pp.3-40; Christyakov, J.Math.Sci. (translation from russian) 93, 1999, 1, pp. 42-133; Grosser et al, Memoirs AMS 153,2001]. The more recent book is Grosser, Kunzinger, Oberguggenberger, Steinbauer, *Geometric Theory of Generalized Functions with applications to General Relativity*, Kluwer, 2001.

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[1] T. Todorov and M. Oberguggenberger are preparing a version of nonlinear generalized functions using the language and technics of non-standard analysis.