MOUNTAIN PASS THEOREMS WITHOUT PALAIS-SMALE CONDITIONS

NATÁLIA MARTINS

We will prove the following lemma:

Lemma. *Let H be a real Hilbert space with norm $\| \cdot \|$. Suppose that $f \in C^1(H, \mathbb{R})$ satisfies the mountain pass geometry with respect to x_1 and x_2, that is, there exist $r \in \mathbb{R}^+$ such that $\| x_1 - x_2 \| > r$ and*

$$\max\{f(x_1), f(x_2)\} \; < \inf_{\|y-x_1\|=r} f(y).$$

Let

$$\Gamma := \{\gamma \in C([0,1], H) : \gamma(0) = x_1 \wedge \gamma(1) = x_2\}$$

and

$$k_1 := \inf_{\gamma\in\Gamma} \max_{t\in[0,1]} f(\gamma(t)).$$

Then

$$\forall \gamma \in {}^\star\Gamma \; \Big[\; \big[\; \gamma({}^\star[0,1]) \subseteq ns({}^\star H) \; \wedge \; \max_{t\in{}^\star[0,1]} f(\gamma(t)) \approx k_1 \; \big] \\ \Rightarrow \; \exists t_0 \in {}^\star[0,1] \; \big[\; f(\gamma(t_0)) \approx k_1 \; \wedge \; \| \, f'(\gamma(t_0)) \, \| \approx 0 \; \big] \; \Big].$$

From this lemma we deduce two new mountain pass theorems which cannot be obtained from the well known classical Mountain Pass Theorem of Ambrosetti-Rabinowitz.

Departamento de Matemática, Universidade de Aveiro, Portugal
E-mail address: nataliam@mat.ua.pt