formula di Wallis

FORMULA DI WALLIS

$$\frac{\pi}{2} = \frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdots\cdots$$

$$I_n = \int_0^{\pi} (\sin x)^n \, dx$$

$$I_0 = \int_0^{\pi} 1 \, dx = \pi$$

$$I_1 = \int_0^{\pi} \sin x \, dx = \left[-\cos x\right]_0^{\pi} = 2$$

$n \geq 2$

$$I_n = \int_0^{\pi} \sin x \cdot (\sin x)^{n-1} \, dx$$

per parti

$$= \cancel{\left[(-\cos x)\cdot(\sin x)^{n-1}\right]_0^{\pi}} - \int_0^{\pi} (-\cos x)\,(n-1)(\sin x)^{n-2} \cos x \, dx$$

$$= (n-1)\int_0^{\pi} \cos^2 x \,(\sin x)^{n-2} \, dx$$

$$= (n-1)\int_0^{\pi} (1-\sin^2 x)(\sin x)^{n-2} \, dx$$

$$I_n = (n-1) I_{n-2} - (n-1) I_n$$

$$I_n + (n-1) I_n = (n-1) I_{n-2}$$

$$n I_n = (n-1) I_{n-2}$$

$$\boxed{I_n = \frac{n-1}{n} I_{n-2}.}$$

$$I_0 = \pi, \quad I_2 = \frac{1}{2}\pi, \quad I_4 = \frac{3}{4}\cdot\frac{1}{2}\pi$$

$$I_6 = \frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\pi$$

$$I_{2n} = \frac{(2n-1)(2n-3)(2n-5)\cdots 1}{(2n)(2n-2)(2n-4)\cdots 2}\pi$$

$$I_{2n} = \frac{(2n-1)!!}{(2n)!!}\pi.$$

$$I_1 = 2, \quad I_3 = \frac{2}{3}\cdot 2, \quad I_5 = \frac{4}{5}\cdot\frac{2}{3}\cdot 2$$

$$I_7 = \frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}\cdot 2 \quad \ldots$$

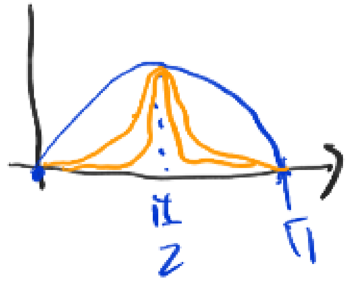
$$I = \frac{(2n)(2n-2)(2n-4)\cdots 2}{[illegible]}\, 2$$

$$I_{2n+1} = \frac{}{(2n+1)(2n-1)(2n-3)\cdots 1}$$

$$= \frac{(2n)!!}{(2n+1)!!}\cdot 2 .$$

<u>Idea</u>: Mostrare che $\dfrac{I_{2n+1}}{I_{2n}} \longrightarrow 1$.

$$I_n = \int_0^\pi (\sin x)^n \, dx$$



I_n è decrescente.

$$\frac{I_n}{I_{n-2}} = \frac{n-1}{n} \longrightarrow 1 .$$

$$\frac{I_{2n+2}}{I_{2n}} \le \frac{I_{2n+1}}{I_{2n}} \le \frac{I_{2n}}{I_{2n}} = 1$$

$$\frac{I_{2n+2}}{I_{2n}} = \frac{n+1}{n+2} \longrightarrow 1 \qquad \frac{I_{2n+1}}{I_{2n}} \longrightarrow 1$$

2 carabinieri.

$$1 \leftarrow \frac{I_{2n+1}}{I_{2n}} = \frac{\dfrac{(2n)!!}{(2n+1)!!} \cdot 2}{\dfrac{(2n-1)!!}{(2n)!!}\, \pi}$$

$$= \frac{2}{\pi} \cdot \frac{\left((2n)!!\right)^2}{(2n+1)!!\,(2n-1)!!}$$

$$\boxed{\frac{(2n!!)^2}{(2n+1)!!\,(2n-1)!!} \longrightarrow \frac{\pi}{2}}$$

$$\frac{(2n)(2n)(2n-2)(2n-2)\dots 2\cdot 2}{(2n+1)(2n-1)(2n-1)(2n-3)(2n-3)\cdot 1 \cdot 1}$$

$\square$

COEFFICIENTE BINOMIALE CENTRALE

$$(2n)!! = \overbrace{(2n)(2n-2)(2n-4)\dots 4\cdot 2}^{n \text{ fattori}}$$

$$(2n)!! = (2n)(2n-2)(2n-4)\cdots 4\cdot 2$$

$$= 2^n\, n(n-1)(n-2)\cdots 2\cdot 1$$

$$= 2^n \cdot n!$$

$$(2n+1)!! = \frac{(2n+1)!}{(2n)!!} = \frac{(2n+1)!}{2^n\, n!}$$

$$\frac{\pi}{2} \leftarrow \frac{((2n)!!)^2}{(2n+1)!!\,(2n-1)!!} \quad (n \to +\infty)$$

$$= \frac{(2^n n!)^2\, 2^n n!\; 2^{n-1}(n-1)!}{(2n+1)!\;(2n-1)!}$$

$$= \frac{(2^n)^4\,(n!)^4}{2\; n\;(2n+1)!\,(2n-1)!}$$

$$= \frac{2^{4n}(n!)^4}{(2n+1)!\,(2n)!}$$

$$= \boxed{\frac{2^{4n}\boxed{(n!)^4}}{\quad} \longrightarrow \frac{\pi}{2}}$$

$$(2n+1)\,\boxed{\left((2n)!\right)^2} \quad <$$

$$\binom{2n}{n} = \frac{(2n)!}{n!\,n!} = \frac{(2n)!}{(n!)^2}$$

$n=0 \rightarrow$ 1
1 1
$n=2 \rightarrow$ 1 2 1
1 3 3 1
$n=4$ 1 4 6 4 1
1 5 10 10 5 1
$n=6$ 1 6 15 20 15 6 1

$$\frac{\left((2n)!\right)^2}{(n!)^4} \sim \frac{2}{\pi}\,\frac{2^{4n}}{2n+1}$$

$$\frac{(2n)!}{(n!)^2} \sim \sqrt{\frac{\cancel{2}}{\pi}}\,\frac{4^n}{\sqrt{\cancel{2}n+\cancel{1}}} = \frac{4^n}{\sqrt{n\cdot\pi}}$$

$$\sqrt{2n+1} \sim \sqrt{2n}$$

$$\boxed{\binom{2n}{n} \sim \frac{4^n}{\sqrt{n\cdot\pi}}}$$

* errore nel video!