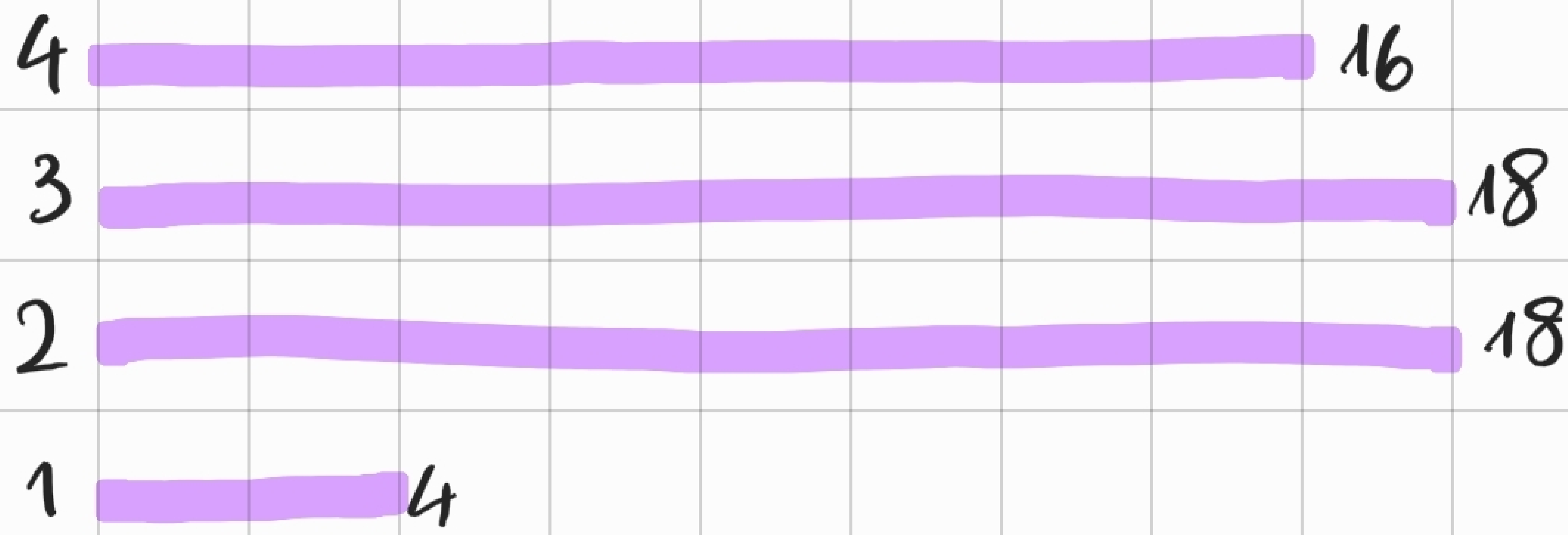
ANALISI MATEMATICA B
LEZIONE 44 – 25.1.2021

test settimanale



$$\sum_{n=1}^{+\infty} \sin \frac{\pi(n^2+1)}{n}$$

$$\sin \frac{\pi(n^2+1)}{n} = \sin\left(n\pi + \frac{\pi}{n}\right)$$

$$= \cancel{\sin(n\pi)} \cos\frac{\pi}{2} + \cos(n\pi)\sin\frac{\pi}{n}$$

$$= (-1)^n \sin\frac{\pi}{n}$$

$$\sum (-1)^n \sin\frac{\pi}{n}$$

Leibniz

$\sin\frac{\pi}{n} \to 0$ per $n \to +\infty$

$\sin\frac{\pi}{n}$ decrescente se $n \geq 2$

è convergente

con regressa (?) condotta.

$$\left|(-1)^n \sin\frac{\pi}{n}\right| = \sin\frac{\pi}{n} \sim \frac{\pi}{n}$$

$$\sum \frac{\pi}{n} = \pi \sum \frac{1}{n} = +\infty$$

la serie non converge assolutamente.

Derivate

derivabile $\Rightarrow$ continua

$$D\,\frac{1}{x} = -\frac{1}{x^2}$$

derivata funzione composta:

$$\big(f(g(x))\big)' = f'(g(x))\,g'(x)$$

$$\begin{bmatrix} \neq \\ f'(g(x)) \end{bmatrix}$$

derivata funzione inversa:

$$\left(f^{-1}\right)'(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\left(f^{-1}\right)'(f(x)) = \frac{1}{f'(x)}$$

<u>Derivata della somma:</u> f, g derivabili in x

$$\to \left[f(x) \pm g(x)\right]' = f'(x) \pm g'(x)$$

$$D(f \pm g) = Df \pm Dg$$

<u>dim</u>

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$(f \pm g)'(x) = \lim_{h \to 0} \frac{\left(f(x+h) \pm g(x+h)\right) - \left(f(x) \pm g(x)\right)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \pm \frac{g(x+h) - g(x)}{h}\right]$$

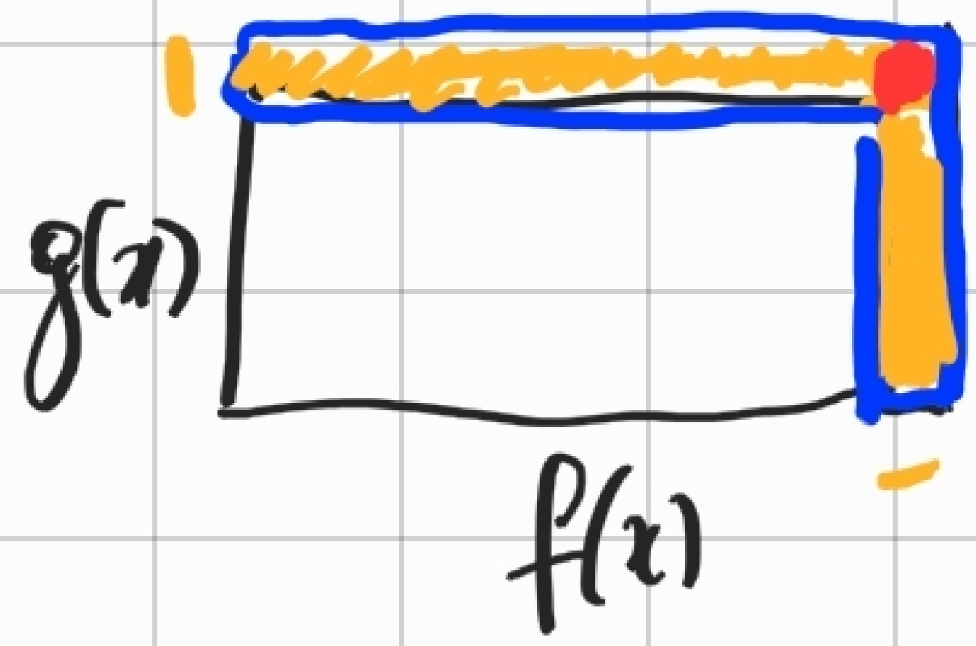
$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \pm \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) \pm g'(x) \qquad \square$$

$$f'(x) = \frac{df}{dx}$$

<u>Derivata del prodotto</u> f, g derivabili in x

$$(f \cdot g)'(x) = f'(x)\, g(x) + f(x) \cdot g'(x)$$



$$\frac{(f\cdot g)(x+h)-(f\cdot g)(x)}{h} = \frac{f(x+h)\cdot g(x+h)-f(x)\cdot g(x)}{h}$$

$$= \frac{f(x+h)\,g(x+h) - f(x+h)\cdot g(x) + f(x+h)\cdot g(x) - f(x)\,g(x)}{h}$$

$$= \frac{f(x+h)\left[g(x+h)-g(x)\right] + \left[f(x+h)-f(x)\right]\cdot g(x)}{h}$$

$$= f(x+h)\cdot \underbrace{\frac{g(x+h)-g(x)}{h}} + \frac{f(x+h)-f(x)}{h}\cdot g(x)$$

f continua $\downarrow$ $\qquad \downarrow \qquad \downarrow$ per $h \to 0$

$$f(x) \cdot g'(x) + f'(x)\cdot g(x)$$

$\square$

Derivata della differenza

$$\begin{cases} \left(f(x)-g(x)\right)' = \left(f(x)+(-g(x))\right)' = f'(x) + (-g)'(x) \\ \left(-g(x)\right)' = \left[h(g(x))\right]' = -1 \cdot g'(x) \\ h(y) = -y \qquad h' = -1 \end{cases}$$

$$(f-g)' = f' - g'$$

<u>Derivata del rapporto</u>

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$\frac{f(x)}{g(x)} = f(x) \cdot h(g(x))$$

con $h(y) = \frac{1}{y} \qquad h'(y) = -\frac{1}{y^2}$

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x) \cdot h(g(x))\right)'$$

$$= f'(x) \cdot h(g(x)) + f(x) \cdot h'(g(x)) \cdot g'(x)$$

$$= \frac{f'(x)}{g(x)} + f(x) \cdot \left(-\frac{1}{g^2(x)}\right) g'(x)$$

$$= \frac{f'(x) \cdot g(x) - f(x)\, g'(x)}{g^2(x)} \qquad \square$$

Esercizio $\quad f(x) = \dfrac{x^2}{1+x}$

$$f'(x) = \frac{(x \cdot x)'(1+x) - x^2 \cdot (1+x)'}{(1+x)^2}$$

$$= \frac{2x(1+x) - x^2 \cdot 1}{(1+x)^2}$$

$$= \frac{2x + 2x^2 - x^2}{(1+x)^2}$$

$$= \frac{2x + x^2}{(1+x)^2}$$

Derivate delle funzioni elementari

costante

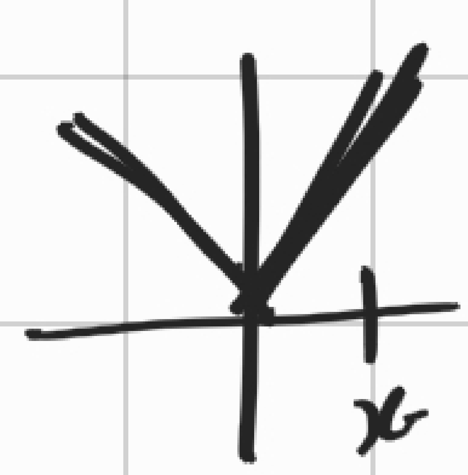
$$\boxed{Dc = 0}$$

lineari:

$$\frac{d}{dx}(m \cdot x + q) = m \qquad f(x) = mx + q$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left(m(\cancel{x}+h)+\cancel{q}\right) - (m\cancel{x}+\cancel{q})}{h} = \frac{mh}{h} = m \downarrow m$$

valore assoluto:



$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

$$D|x| = \begin{cases} 1 & x > 0 \\ \nexists & x = 0 \\ -1 & x < 0 \end{cases} = \frac{x}{|x|}$$

Potenza a esponente intero

$$D x^n = \underbrace{n x^{n-1}} \qquad n \in \mathbb{Z},\ n \neq 0$$

$n \in \mathbb{N}$ $n > 0$ per induzione $D x^1 = 1 = 1 \cdot x^0$ ✓

$$D x^{n+1} = D(x^n \cdot x) = n x^{n-1} \cdot x + x^n \cdot 1$$

ipotesi induttiva

$$= (n+1) x^n \quad ✓$$

$\boxed{n<0}$

$-n = |n|$

$$D x^n = D \frac{1}{x^{|n|}} = - \frac{1}{x^{2|n|}} \cdot |n| x^{|n|-1}$$

$$= - \frac{1}{x^{-2n}} \cdot (-n) \cdot x^{-n-1} = n \cdot x^{-n-1+2n}$$

$$= n \cdot x^{n-1}.$$

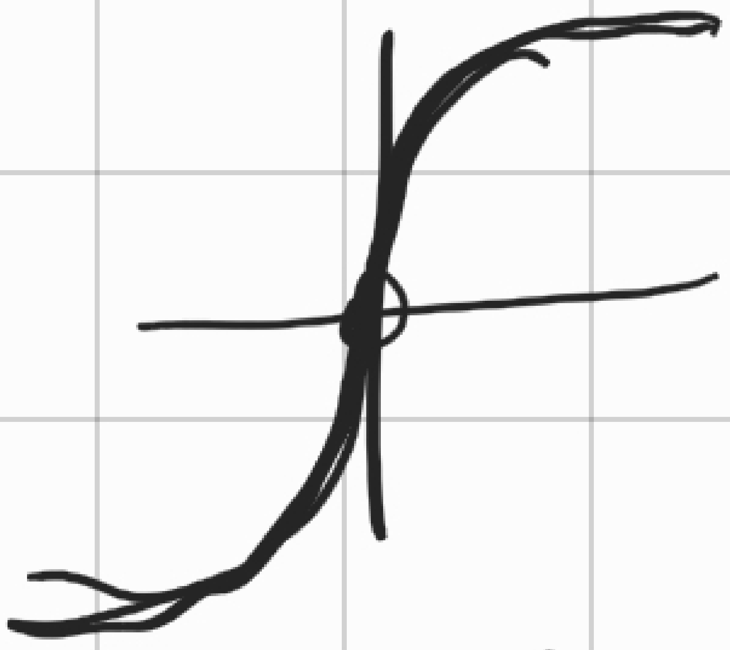
$$\boxed{D \frac{1}{f(x)} = - \frac{f'(x)}{f^2(x)}}$$

Radici: $x \neq 0$ $\quad [\, f(x) = x^n \quad f^{-1}(y) = \sqrt[n]{y} \,]$

$$D \sqrt[n]{x} = \frac{1}{n (\sqrt[n]{x})^{n-1}}$$

$$\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$$

$$\left[D\, x^{\frac{1}{n}} = \frac{1}{n} x^{-\frac{n-1}{n}} = \frac{1}{n} \cdot x^{\frac{1}{n} - 1} \right]$$



$$\boxed{D\sqrt{x} = \frac{1}{2\sqrt{x}}} \quad x \neq 0$$

Esponenziale:

$$De^x = e^x$$

$$f(x) = e^x$$

$$f'(0) = \lim_{h \to 0} \boxed{\frac{e^h - 1}{h}} = 1$$

$$= e^0$$

$$\frac{e^{x+h} - e^x}{h} = \frac{e^x e^h - e^x}{h} = e^x \frac{e^h - 1}{h}$$

$$\Big\downarrow \; h \to 0$$

$$e^x \, \square$$

$$Da^x = De^{x \cdot \ln a} = e^{x \cdot \ln a} \cdot \ln a$$

$$= \ln a \cdot a^x$$

Logaritmi:

$$f(y) = e^y, \quad f^{-1}(x) = \ln x$$

$$\boxed{D \ln x = \frac{1}{e^{\ln x}} = \frac{1}{x}}$$

$x > 0$ / se $x < 0$

$$D \ln(-x) = \frac{1}{-x} \cdot (-1) = \frac{1}{x}$$

$$D \ln |x| = \frac{1}{x} \qquad \forall x \neq 0$$

Potenze con esponenti reale:

$$D x^{\alpha} = D e^{\alpha \cdot \ln x}$$

$$= e^{\alpha \cdot \ln x} \cdot \alpha \cdot \frac{1}{x} = x^{\alpha} \cdot \alpha \cdot \frac{1}{x}$$

$$\boxed{D x^{\alpha} = \alpha \cdot x^{\alpha-1}} \qquad \begin{array}{l} \forall \alpha \in \mathbb{R} \\ x>0 \end{array}$$

Funzioni trigonometriche

$$\lim_{h\to 0} \frac{\sin h - \sin 0}{h} = 1$$

$$\lim_{h\to 0} \frac{1-\cos h}{h} = 0$$

$$\boxed{D \sin x = \cos x}$$

$$\frac{\sin(x+h) - \sin x}{h} = \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \cos x \cdot \underbrace{\frac{\sin h}{h}}_{\downarrow} + \sin x \cdot \underbrace{\frac{\cos h - 1}{h}}_{\downarrow} \qquad \text{per } h \to 0$$

$$= \cos x \cdot 1 + \sin x \cdot 0$$

$$= \cos x .$$

$$\boxed{D\cos x = -\sin x}$$

$$\frac{\cos(x+h)-\cos x}{h} = \frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \cos x\,\frac{\cos h - 1}{h} - \sin x\,\frac{\sin h}{h}$$

per $h \to 0$

$$\cos x \cdot 0 - \sin x \cdot 1$$

$$= -\sin x$$

$$D\,\mathrm{tg}\,x = D\frac{\sin x}{\cos x} = \frac{\cos x\cdot\cos x - \sin x\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= 1 + \mathrm{tg}^2 x$$

inverse:

$x \in (-1, 1)$

$\arcsin x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\downarrow$

$\cos x > 0$

$$\boxed{D \arcsin x = \frac{1}{\cos(\arcsin x)}}$$

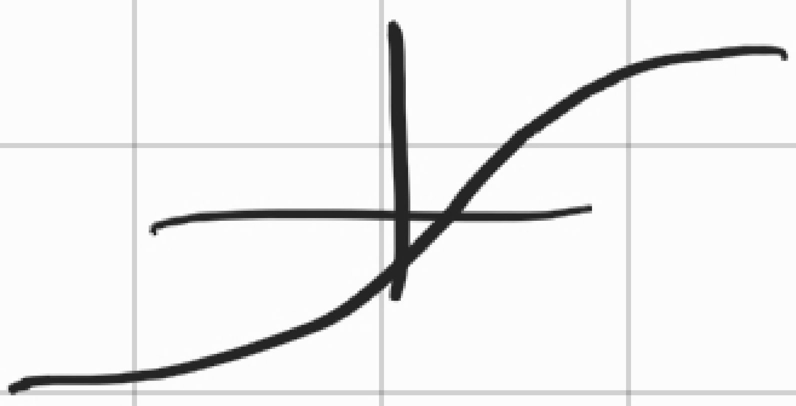
$$= \frac{1}{+\sqrt{1-\sin^2(\arcsin x)}} = \boxed{\frac{1}{\sqrt{1-x^2}}}$$

$$\left[\begin{array}{ll} \cos^2 + \sin^2 = 1 & \cos^2 = 1 - \sin^2 \\ & \cos = \pm\sqrt{1-\sin^2} \end{array}\right]$$

$$\boxed{D \arccos x = \frac{1}{-\sqrt{1-x^2}}}$$

$$\arccos x = \frac{\pi}{2} - \arcsin x$$

$$D \operatorname{arctg} x = \frac{1}{1+\operatorname{tg}^2(\operatorname{arctg} x)}$$

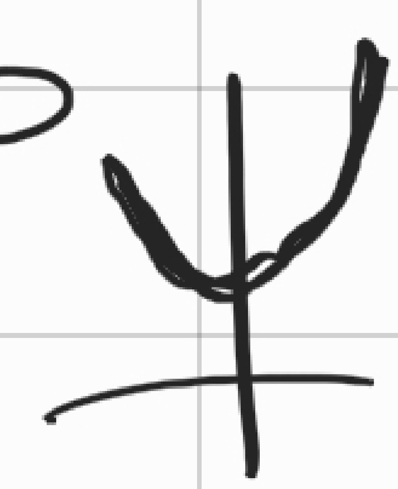
 $$= \frac{1}{1+x^2} \quad \leftarrow \forall x \in \mathbb{R}$$

$\square$

Funzioni iperboliche

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} > 0$$



$$D \sinh x = \frac{e^x + e^{-x}}{2} = \cosh x$$

$$D \cosh x = \frac{e^x - e^{-x}}{2} = \sinh x$$

$$\begin{tikzcd} e^x \arrow[loop below, "D"] \end{tikzcd} \qquad \begin{tikzcd} \sinh x \arrow[d, shift right, "D"'] \\ \cosh x \arrow[u, shift right, "D"'] \end{tikzcd} \qquad \begin{tikzcd} \sin x \arrow[d, "D"'] & -\cos x \arrow[l, "D"'] \\ \cos x \arrow[r, "D"] & -\sin x \arrow[u, "D"'] \end{tikzcd}$$

$$D \operatorname{settsinh} x = \frac{1}{\cosh(\operatorname{settsinh} x)}$$

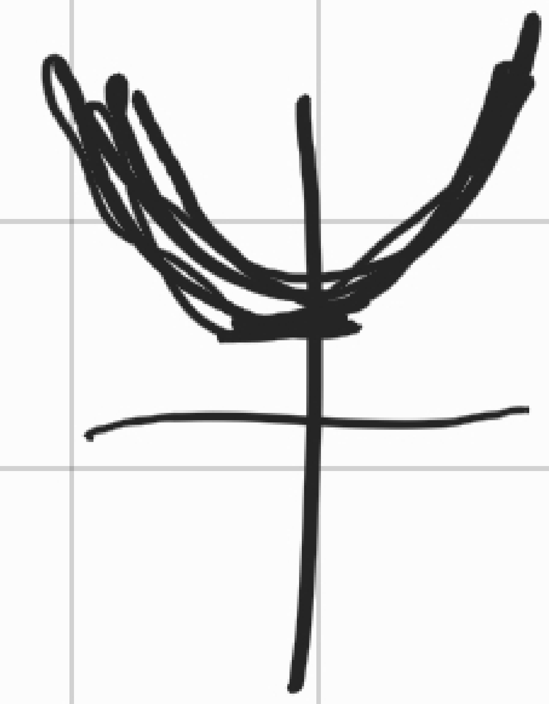
$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh^2 x = 1 + \sinh^2 x$$

$$\cosh x = \sqrt{1 + \sinh^2 x}$$

$$D \operatorname{settsinh} x = \frac{1}{\sqrt{x^2 + 1}}$$

$$D\,\text{sett}\cosh x = \frac{1}{\sqrt{x^2-1}}$$



Esercizio $D\,\text{sett}\,\text{tgh}\, x = ?$

Esercizio Calcolare la derivata.

$$f(x) = \sqrt{1+\cos x}$$

$$f'(x) = ?$$

$$f'(x) = \frac{-\sin x}{2\sqrt{1+\cos x}}$$

$$\left(y^{1/2}\right)' = \frac{1}{2} y^{-\frac{1}{2}}$$

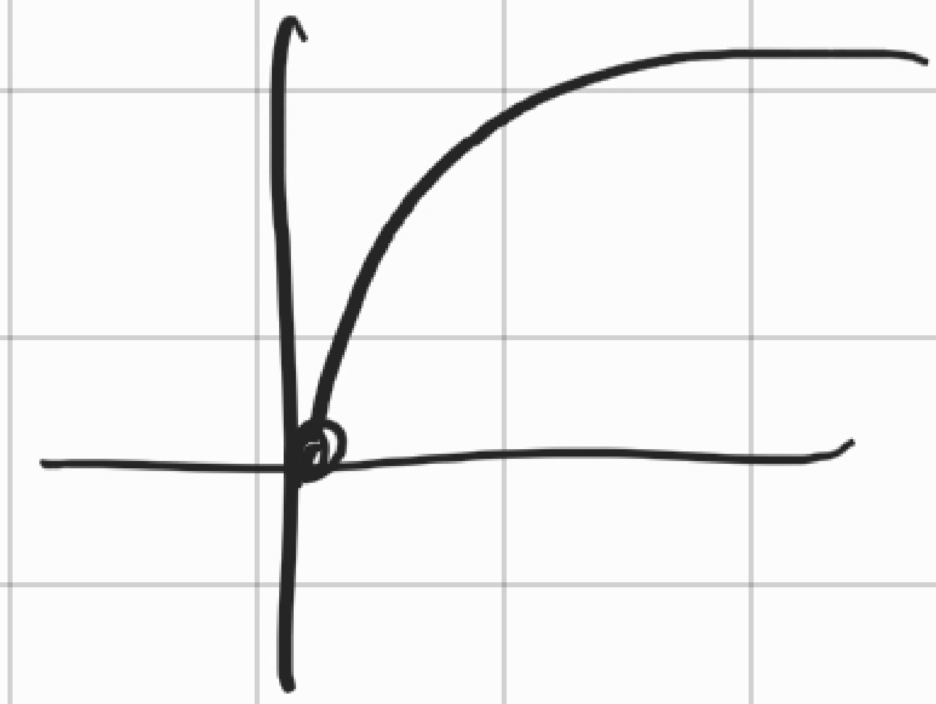
$$D\sqrt{y} = \frac{1}{2\sqrt{y}} \qquad y \neq 0$$

$$D\sqrt{g(x)} = \frac{g'(x)}{2\sqrt{g(x)}}$$

Esercizio

$$f(x) = \frac{e^x \cdot \cos\sqrt{x}}{\sqrt{1+\text{tg}^2 x}} \leftarrow$$

$$f'(x) = \frac{\left(e^x \cdot \cos\sqrt{x} + e^x\left(-\sin\sqrt{x}\right)\frac{1}{2\sqrt{x}}\right)\sqrt{1+tg^2x} - e^x\cos\sqrt{x}\cdot\frac{2\, tg\, x\cdot(1+tg^2x)}{2\sqrt{1+tg^2x}}}{1+tg^2x}$$



Esercizio

$$\boxed{f(x) = \sqrt{1-\cos x}}$$

$$1-\cos x \geq 0 \quad \forall x$$

$$f: \mathbb{R} \to \mathbb{R}$$

$$f'(x) = \begin{cases} \dfrac{\sin x}{2\sqrt{1-\cos x}} & \cos x \neq 1 \\ ? & \cos x = 1 \end{cases}$$

Non posso dire che non esiste.