ANALISI MATEMATICA B
LEZIONE 62 - 8.3.2021

test settimanale:

punti: 3 — 14

2 — 11

1 — 2

0 — 4

Es 2 $$f(x) = \int_0^{x^2} e^{-t^2}\, dt$$

Calcolare $f'(x)$.

$$\boxed{F(x) = \int_0^{x} e^{-t^2}\, dt}$$

$$F'(x) = \underline{e^{-x^2}}$$

$$f(x) = F(x^2)$$

$$f'(x) = F'(x^2) \cdot 2x$$

$$= e^{-x^4} \cdot 2x \qquad \square$$

In generale $$G(x) = \int_{a(x)}^{b(x)} f(t)\, dt$$

$$= \int_{x_0}^{b(x)} f(t)\, dt - \int_{x_0}^{a(x)} f(t)\, dt$$

$$= F(b(x)) - F(a(x))$$

$$F' = f$$

$$G'(x) = \ldots$$

$\square$

Ricerca delle primitive

$$\int f = D^{-1}(\{f\})$$

① integrali immediati

$$D\,\frac{x^{p+1}}{p+1} = x^p \qquad \int x^p\,dx \ni \frac{x^{p+1}}{p+1} \quad \boxed{p \neq -1}$$

$$D \ln|x| = \frac{1}{x} \qquad \int \frac{1}{x}\,dx \ni \ln|x|$$

Es $\int_a^b \frac{1}{x}\,dx$ = ha senso (per ora)

solo se $a, b > 0$

oppure se $a, b < 0$

$$\int_1^2 \frac{1}{x}\,dx = \left[\ln x\right]_1^2 = \ln 2$$

$$D e^x = e^x \qquad \int e^x\,dx \ni e^x.$$

$$D \sin x = \cos x \qquad \int \cos x \, dx = \sin x$$

$$D(-\cos x) = \sin x \qquad \int \sin x \, dx = -\cos x$$

$$D \operatorname{arctg} x = \frac{1}{1+x^2} \qquad \int \frac{1}{1+x^2} \, dx = \operatorname{arctg} x$$

$$D \arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x$$

$$D \operatorname{settsinh} x = \frac{1}{\sqrt{1+x^2}} \qquad \int \frac{1}{\sqrt{1+x^2}} \, dx = \operatorname{settsinh} x = \ln\left(x+\sqrt{x^2+1}\right)$$

$$D \operatorname{settcosh} x = \frac{1}{\sqrt{x^2-1}} \qquad \int \frac{1}{\sqrt{x^2-1}} \, dx = \operatorname{settcosh} x = \ln\left(x+\sqrt{x^2-1}\right)$$

Regole di integrazione

$$\int \left(\lambda f(x) + \mu g(x)\right) dx = \lambda \int f(x) \, dx + \mu \int g(x) \, dx$$

$$H = \lambda \cdot F + \mu G$$

$$F' = f \qquad G' = g$$

$$DH = \lambda F' + \mu G' = \lambda f + \mu g$$

$$V \xrightarrow{L} W$$

$$V/_{\ker L} \xrightarrow{\tilde{L}} W$$

$$\boxed{\begin{array}{c} \ker L = \{0\} \\ \Updownarrow \\ L \text{ iniettivo} \end{array}}$$

<u>Es</u>

$$\int (x+1)\cdot(x^2+1)\,dx$$

$$= \int \left(x^3 + x^2 + x + 1 \right) dx$$

$$= \int x^3 dx + \int x^2 dx + \int x\,dx + \int 1\,dx$$

$$= \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x$$

$$\boxed{\int f = \varnothing}$$

$$\int (f - f) = \int 0 = \{ c : c \in \mathbb{R} \}$$

funzione

$$c(x) = c$$

$$\cancel{=}$$

$$\int f - \int f$$

$$\int 0 = \int 0\cdot f \cancel{=} 0 \cdot \int f = \varnothing$$

funzione composta (F, g derivabili)

$$\underbrace{\left(F(g(x))\right)'}_{} = \underbrace{F'(g(x))\cdot g'(x)}_{}$$

Se $F \in \int f$

$$\to F\circ g \in \int f(g(x))\, g'(x)\, dx$$

$$F(g(x)) = \left[\int f(y)\, dy\right]_{\underline{y=g(x)}} \subseteq \int f(g(x))\, g'(x)\, dx$$

$$\boxed{\int f(g(x))\, g'(x)\, dx \supseteq \left[\int f(y)\, dy\right]_{y=g(x)}}$$

integrazione per sostituzione (cambio di variabile) diretta

MNEMONICO

$$\begin{cases} y = g(x) \\ \underbrace{dy = g'(x)\, dx}_{} \end{cases} \leftarrow \underbrace{\frac{dy}{dx} = \frac{d\,g(x)}{dx} = g'(x)}_{}$$

$$\underbrace{\int f(x)\, dx}_{} \leftarrow \underbrace{\sum (x_{k+1} - x_k)\cdot \sup_{\inf} f}_{\int \quad \Delta x}$$

Es
$$\int (e^x)^2\,dx = \int e^{2x}\,dx$$
$$= \frac{1}{2}\int e^{2x}\cdot \underline{2\,dx} = \frac{1}{2}\left[\int e^y\,dy\right]_{y=2x}$$

$y = 2x$ $\qquad g(x) = 2x$
$dy = 2\cdot dx$

$$= \frac{1}{2}\, e^y \Big|_{y=2x} = \frac{1}{2}\, e^{2x} \qquad \square$$

Es
$$\int \cos^2 x\,dx =$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1-\cos^2 x)$$
$$= \underline{2\cos^2 x} - 1$$

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\int \cos^2 x\,dx = \int\left[\frac{1}{2} + \frac{\cos(2x)}{2}\right]dx$$
$$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \cos(2x)\,dx$$
$$= \frac{x}{2} + \frac{1}{4}\int \cos(2x)\cdot 2\,dx$$

$$\left\{\begin{array}{l} y = 2x \\ dy = 2\,dx \end{array}\right. \quad \text{sostituiamo}$$

$$= \frac{x}{2} + \frac{1}{4}\int \cos y \, dy$$

$$= \frac{x}{2} + \frac{1}{4} \sin y \Big|_{y=2x}$$

$$= \frac{x}{2} + \frac{1}{4}\sin(2x).$$

$$= \frac{x}{2} + \frac{1}{2}\sin x \cos x = \frac{x + \sin x \cos x}{2}$$

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(2) Sostituzione negli integrali definiti

$$\Big[\int f(g(x))\, g'(x)\, dx\Big]_a^b = \Big[\Big[\int f(y)\, dy\Big]_{y=g(x)}\Big]_a^b$$

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(y)\, dy$$

$$\left\{\begin{array}{l} y = g(x) \\ dy = g'(x)\, dx \\ x = a \iff y = g(a) \\ x = b \iff y = g(b) \end{array}\right.$$

Es

$$\boxed{\int_0^\pi \cos^2 x\,dx} = \int_0^\pi \left[\frac{1}{2} + \frac{\cos 2x}{2}\right] dx$$

$$= \frac{1}{2}\int_0^\pi dx + \frac{1}{4}\int_0^\pi \cos(2x)\cdot 2\cdot dx$$

$dg(x) = g'(x)\,dx$

$$\left[= \frac{1}{2}\int_0^\pi dx + \frac{1}{4}\int_{0\,?}^{2\pi\,?} \cos(2x)\cdot d(2x)\right]$$

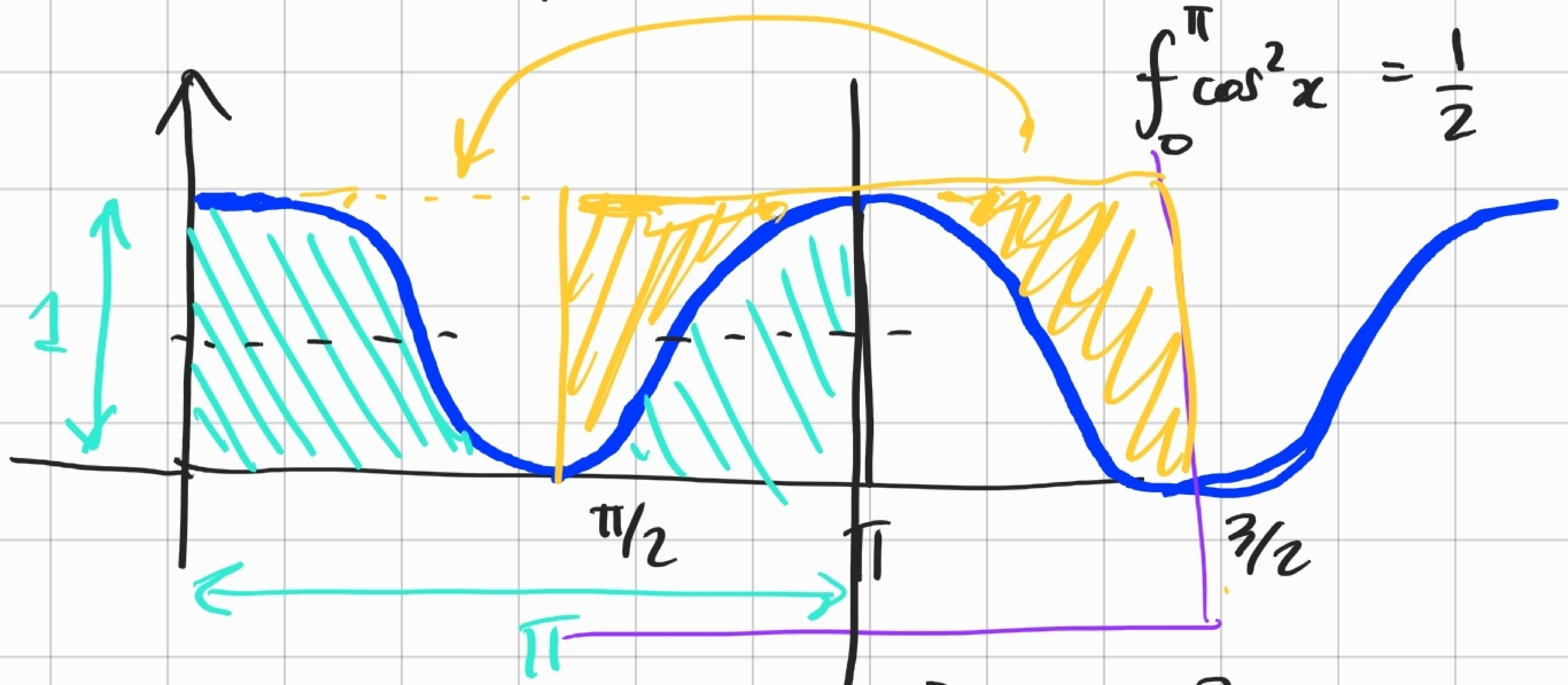
$\begin{cases} y = 2x \\ dy = 2dx \end{cases}$

$$= \frac{1}{2}\left[x\right]_0^\pi + \frac{1}{4}\int_0^{2\pi} \cos y \; dy$$

$$= \frac{\pi}{2} + \frac{1}{4}\cdot\left[\sin y\right]_0^{2\pi}$$

$$= \frac{\pi}{2} + \frac{1}{4}\left(\sin(2\pi) - \sin 0\right) = \frac{\pi}{2}$$

$\int_0^\pi \cos^2 x = \frac{1}{2}$



$$\underbrace{1 - \cos^2 x = \sin^2 x}$$

$$\int_0^{\pi} \cos^2 x \cdot dx \overset{?}{=} \int_0^{\pi} \sin^2 x \, dx$$

$$\cos\left(y - \frac{\pi}{2}\right) = \sin y$$

$$\begin{cases} y = \frac{\pi}{2} + x \\ dy = dx \end{cases}$$

$$\int_0^{\pi} \cos^2 x \cdot dx = \int_0^{\pi} \left(\cos\left(\frac{\pi}{2} + x - \frac{\pi}{2}\right)\right)^2 dx$$

$$= \int_{\pi/2}^{\frac{3}{2}\pi} \cos^2\left(y - \frac{\pi}{2}\right) dy = \int_{\frac{\pi}{2}}^{3/2\,\pi} \sin^2 y \, dy$$

$$\begin{cases} y = \frac{\pi}{2} + x \\ dy = dx \end{cases}$$

$$= \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \, dx + \int_{\pi}^{3/2\,\pi} \sin^2 x \, dx$$

$$y = x - \pi$$

$$= \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \, dx + \int_{\pi}^{3/2\,\pi} \sin^2(\underbrace{x - \pi} + \pi) \, dx$$

$$= \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \, dx + \int_0^{\pi/2} \sin^2(y + \pi) \, dy$$

$\sin(y+\pi) = -\sin(y)$

$$= \int_{\frac{\pi}{2}}^{\pi} \sin^2 x \, dx + \int_0^{\pi/2} \sin^2 x \, dx$$

$$= \int_0^{\pi} \sin^2 x \, dx .$$

$g(x) = x^2$

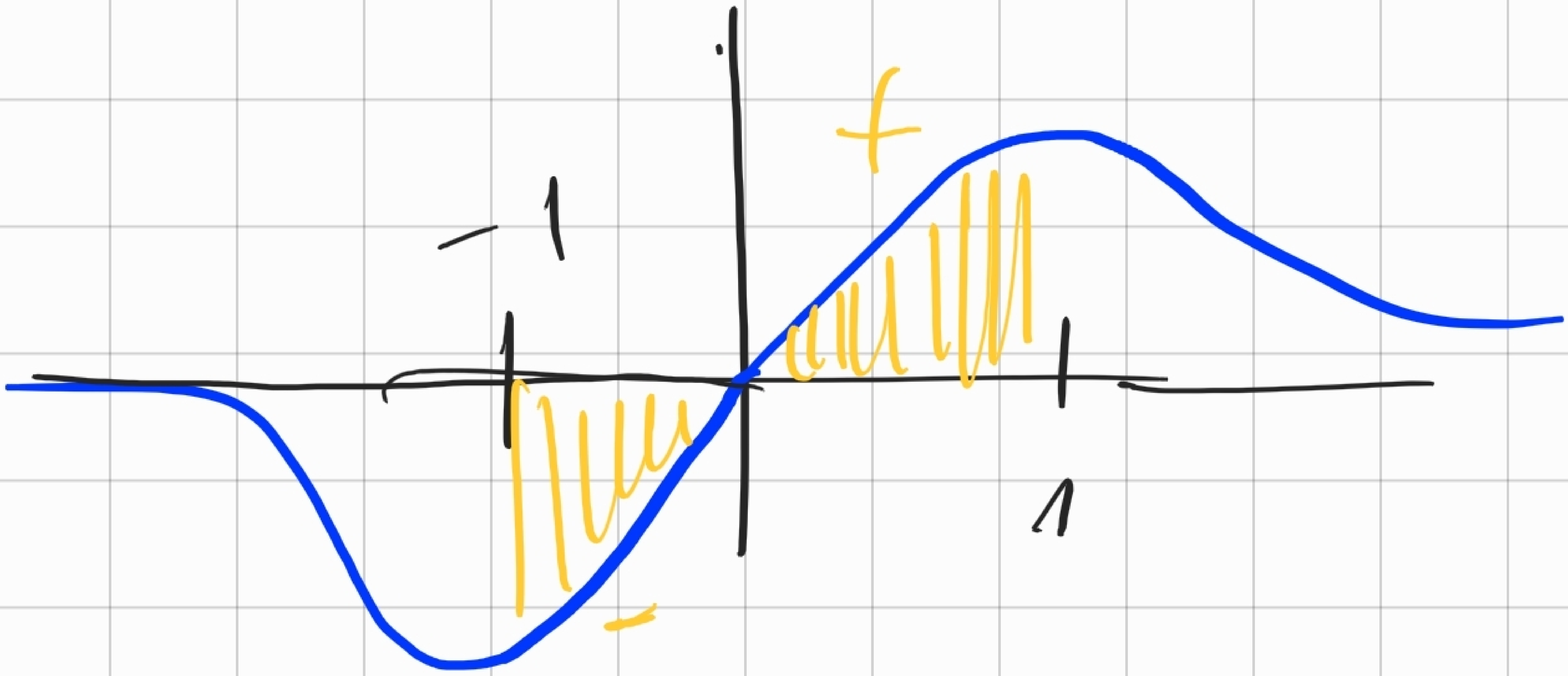
$$\boxed{\int f(g(x)) \cdot g'(x) \, dx}$$

Es
$$\int x \cdot e^{x^2} dx = \frac{1}{2} \int e^{x^2} \cdot 2x \, dx$$

$y = x^2$
$dy = 2x \, dx$

$$= \frac{1}{2} \int e^y dy = \frac{e^y}{2} = \frac{e^{x^2}}{2}$$

$$\int_{-1}^{1} x \cdot e^{x^2} dx \overset{?}{=} \frac{1}{2} \int_1^1 e^y dy = 0$$



$\square$