

Algebra lineare - Esercizi del 5/11/08

Calcolare le dimensioni di $X, Y, X \cap Y, X+Y$ verificando le formule di Grassmann:

$$(1) X = \text{Span} \left(\begin{pmatrix} -1 \\ 4 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \\ 3 \end{pmatrix} \right) \subset \mathbb{R}^4$$

$$Y = \text{Span} \left(\begin{pmatrix} 3 \\ 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -9 \\ 2 \\ 0 \\ 8 \end{pmatrix} \right) \subset \mathbb{R}^4$$

$$(2) X = \text{Span} \left(\begin{pmatrix} 1 \\ 2 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ -9 \\ 16 \end{pmatrix} \right) \subset \mathbb{R}^4$$

$$Y = \left\{ x \in \mathbb{R}^4 : \begin{matrix} x_1 + 2x_3 + x_4 = 0 \\ 4x_1 + 5x_2 + 3x_3 = 0 \end{matrix} \right\}$$

$$(3) X = \left\{ x \in \mathbb{R}^4 : \begin{matrix} 2x_1 - x_2 + 3x_3 + 4x_4 = 0 \\ 3x_1 + 2x_2 - 5x_3 - x_4 = 0 \\ -5x_1 - 8x_2 + 21x_3 + 11x_4 = 0 \end{matrix} \right\}$$

$$Y = \left\{ x \in \mathbb{R}^4 : \begin{matrix} x_1 + 3x_2 - 2x_3 + 5x_4 = 0 \\ 7x_1 + 9x_3 - 3x_4 = 0 \end{matrix} \right\}$$

$$(4) X = \text{Span} \left(\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 3 \end{pmatrix} \right) \subset M_{2 \times 3}(\mathbb{R})$$

$$Y = \left\{ A \in M_{2 \times 3}(\mathbb{R}) : a_{11} = a_{12} = a_{13}, 2a_{21} - 5a_{22} + a_{23} = 0 \right\}$$

$$(5) X = \text{Span} (1-t+t^2+t^3, 2+t-2t^3+3t^4) \subset \mathbb{R}_{\leq 4}[t]$$

$$Y = \left\{ p(t) \in \mathbb{R}_{\leq 4}[t] : p(-1) = 0, p'''(\frac{1}{6}) = 0 \right\}$$