

Algebra lineare - Esercizi del 6/11/08

Esibire basi di nucleo e immagine delle seguenti $f: V \rightarrow W$ lineari, verificando le formule delle dimensioni.

$$(1) \quad f: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \quad f(x) = \begin{pmatrix} x_1 - 2x_2 + x_3 \\ 2x_1 + 4x_2 - 3x_3 \end{pmatrix}$$

$$(2) \quad f: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \quad f(x) = \begin{pmatrix} x_1 - 2x_2 \\ 2x_1 + 3x_2 \\ -x_1 + 4x_2 \end{pmatrix}$$

$$(3) \quad f_A: \mathbb{R}^3 \rightarrow \mathbb{R}^4 \quad A = \begin{pmatrix} -1 & 2 & -4 \\ 2 & -3 & 7 \\ 3 & 1 & 5 \\ 1 & 4 & -2 \end{pmatrix}$$

$$(4) \quad f_A: \mathbb{R}^3 \rightarrow \mathbb{R}^4 \quad A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 2 & -3 & 2 \end{pmatrix}$$

$$(5) \quad f_A: \mathbb{R}^4 \rightarrow \mathbb{R}^3 \quad A = \begin{pmatrix} -1 & 2 & 0 & -5 \\ 2 & -3 & 1 & 8 \\ 3 & 1 & 7 & 4 \end{pmatrix}$$

$$(6) \quad f: M_{m \times m}(\mathbb{R}) \rightarrow M_{m \times m}(\mathbb{R}) \quad f(A) = A + {}^t A$$

$$(7) \quad f: M_{m \times m}(\mathbb{R}) \rightarrow M_{m \times m}(\mathbb{R}) \quad f(A) = A - {}^t A$$

$$(8) \quad f: M_{2 \times 2}(\mathbb{R}) \rightarrow \mathbb{R}^3 \quad f((a_{ij})) = \begin{pmatrix} a_{11} - a_{12} + a_{22} \\ a_{21} + a_{12} - a_{22} \\ a_{11} + a_{21} \end{pmatrix}$$

$$(9) \quad f: \mathbb{R}^3 \rightarrow M_{2 \times 2}(\mathbb{R}) \quad f(x) = \begin{pmatrix} x_1 + x_2 & x_2 - x_3 \\ x_1 - x_3 & x_2 + x_3 \end{pmatrix}$$

$$(10) \quad f: \mathbb{R}_{\leq d}[t] \rightarrow \mathbb{R}_{\leq d}[t] \quad f(p(t)) = p'(t)$$

$$(11) \quad f: \mathbb{R}_{\leq 3}(t) \rightarrow \mathbb{R}^2 \quad f(p(t)) = \begin{pmatrix} p(1) - p'(-1) \\ p(-1) + p''(1) \end{pmatrix}$$

$$(12) \quad f: \mathbb{R}^4 \rightarrow \mathbb{R}_{\leq 3}[t]$$

$$f(x) = (2x_1 - x_2 + x_3) + (x_1 + 3x_2 - x_3)t + (3x_1 - 5x_2 + 3x_3)t^2$$

$$(13) \quad f: \mathcal{F}(\{a, b, c\}, \mathbb{R}) \rightarrow \mathbb{R}$$

$$\begin{aligned} (f(x))(a) &= x(a) - 3x(b) + 2x(c) \\ (f(x))(b) &= 2x(a) + x(b) - x(c) \\ (f(x))(c) &= 5x(a) - 8x(b) + 5x(c) \end{aligned}$$

$$(14) \quad f: \mathcal{F}(\{a, b, c\}, \mathbb{R}) \rightarrow \mathbb{R}^2$$

$$f(x) = \begin{pmatrix} 2x(a) - 3x(b) + x(c) \\ 4x(a) + 5x(b) + 2x(c) \end{pmatrix}$$

$$(15) \quad f: \mathbb{R}^4 \rightarrow \mathcal{F}(\{a, b, c\}, \mathbb{R})$$

$$\begin{aligned} (f(x))(a) &= 3x_1 - 7x_2 + 4x_3 - 5x_4 \\ (f(x))(b) &= -2x_1 + 4x_2 - 3x_3 + x_4 \\ (f(x))(c) &= 5x_1 - 13x_2 + 6x_3 - 13x_4 \end{aligned}$$