

Foglio 2

1) a. $f: \mathbb{Z} \rightarrow \mathbb{Z}$
 $x \mapsto 2x - 7$

SURJETTIVA? SÌ A $y \in \mathbb{Z}$: $\exists x \in \mathbb{Z}$: $f(x) = y \Rightarrow 2x - 7 = y$
NO!
PAR
DISPARI

QUINDI NON INVERTIBILE!

b. $f: \mathbb{Q} \rightarrow \mathbb{Q}$
 $x \mapsto 2x - 7$

SURJETTIVA? SÌ A $y \in \mathbb{Q}$: $\exists x \in \mathbb{Q}$: $f(x) = y \Rightarrow 2x - 7 = y$

$$\Rightarrow x = \frac{y+7}{2} \quad \checkmark$$

INIETTIVA? $x_1, x_2 \in \mathbb{Q}$: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$$2x_1 - 7 = 2x_2 - 7 \Rightarrow x_1 = x_2 \quad \checkmark$$

INVERSA $g: \mathbb{Q} \rightarrow \mathbb{Q}$
 $y \mapsto \frac{y+7}{2}$

$$c) X = \{n \in \mathbb{N} : n \leq 6\}$$

$$f: X \rightarrow X$$

$x \mapsto$ RESTO DELLA DIVISIONE DI $4x+1 : 7$

x	$f(x)$
0	1
1	5
2	2
3	6
4	3
5	0
6	4

INIETTIVA + SURIETTIVA (BIETTIVA)

$$d) X = \{n \in \mathbb{N} : n \leq 7\}$$

$$f: X \rightarrow X$$

$x \mapsto$ RESTO DELLA DIVISIONE ~~PER~~ $4x+1 : 8$

x	$f(x)$
0	1
1	5
2	1
3	
4	
...	

$$f(0) = f(2) \text{ MA } 0 \neq 2$$

\Rightarrow f NON È INIETTIVA (NE SURIETTIVA)

\Rightarrow f NON È INVERTIBILE

$$e) f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\}$$

$$x \mapsto \frac{1}{2x}$$

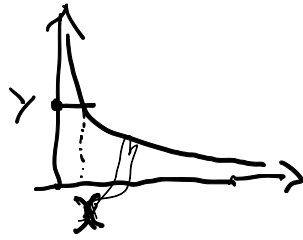
SURIETTIVA? $y \in (\mathbb{R} \setminus \{0\}) \quad \exists x \in \mathbb{R} \setminus \{0\} : f(x) = \frac{1}{2x} = y$

$$\Rightarrow x = \frac{1}{2y} \quad \checkmark$$

INIETTIVA $f(x_1) = f(x_2) \Rightarrow \frac{1}{2x_1} = \frac{1}{2x_2} \Rightarrow x_1 = x_2 \quad \checkmark$

INVERSA $g: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\}$
 $y \mapsto \frac{1}{2y}$

f) $f: \mathbb{R}_{>0} \rightarrow \mathbb{R}_{>0}$
 $x \mapsto \frac{1}{2x}$



g) $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto x^{10}$

$(f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0})$ INIETTIVA X
 SURIETTIVA ✓
 $(f: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0})$ BIETTIVA

SURIETTIVA X $y = -1 \in \mathbb{R} \quad \nexists x: x^{10} = -1$

INIETTIVA X $f(x) = f(-x)$

h) $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto x^{11}$

INVERTIBILE $g: \mathbb{R} \rightarrow \mathbb{R}$
 $y \mapsto \sqrt[11]{y}$

i) $f: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$

$v \mapsto \text{AREA DEL CERCHIO DI RAGGIO } v = \pi v^2$

INVERTIBILE $g: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$

$A \mapsto \sqrt{\frac{A}{\pi}}$

j) $f: \mathbb{Z} \rightarrow \mathbb{Z}$

$x \mapsto \begin{cases} x & \text{SE } x \text{ È PARI} \\ x+7 & \text{SE } x \text{ È DISPARI} \end{cases}$

x PARI $f(x) = x$ PARI
 x DISPARI $f(x) = x+7$ PARI

$$\left. \begin{array}{l} f(-7) = -7+7=0 \\ f(0) = 0 \end{array} \right\} f \text{ NON INIETTIVA} \Rightarrow \text{NON INVERTIBILE.}$$

Es 2

$$a) \binom{n-2}{k+1} + \binom{n-1}{k} + \binom{n-2}{k} = \binom{n}{n-k-1}$$

$$\begin{array}{l} \bullet \binom{n}{k} = \binom{n}{n-k} \quad \textcircled{A} \\ \bullet \binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k} \quad \textcircled{B} \end{array} \quad \left. \vphantom{\begin{array}{l} \bullet \binom{n}{k} = \binom{n}{n-k} \\ \bullet \binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k} \end{array}} \right\} \textcircled{A}$$

$$\begin{aligned} \binom{n}{n-k-1} &= \binom{n}{n-(n-k-1)} \\ &= \binom{n}{n-n+k+1} \end{aligned}$$

$$\binom{n-2}{k+1} + \binom{n-1}{k} + \binom{n-2}{k} = \binom{n}{n-n+k+1}$$

USO \textcircled{B} $\binom{n-2}{k+1} + \binom{n-2}{k} = \binom{n-1}{k+1}$

$$\binom{n-1}{k+1} + \binom{n-1}{k} = \binom{n}{k+1}$$

USO \textcircled{B} $\binom{n}{k+1} = \binom{n}{k+1}$ ✓