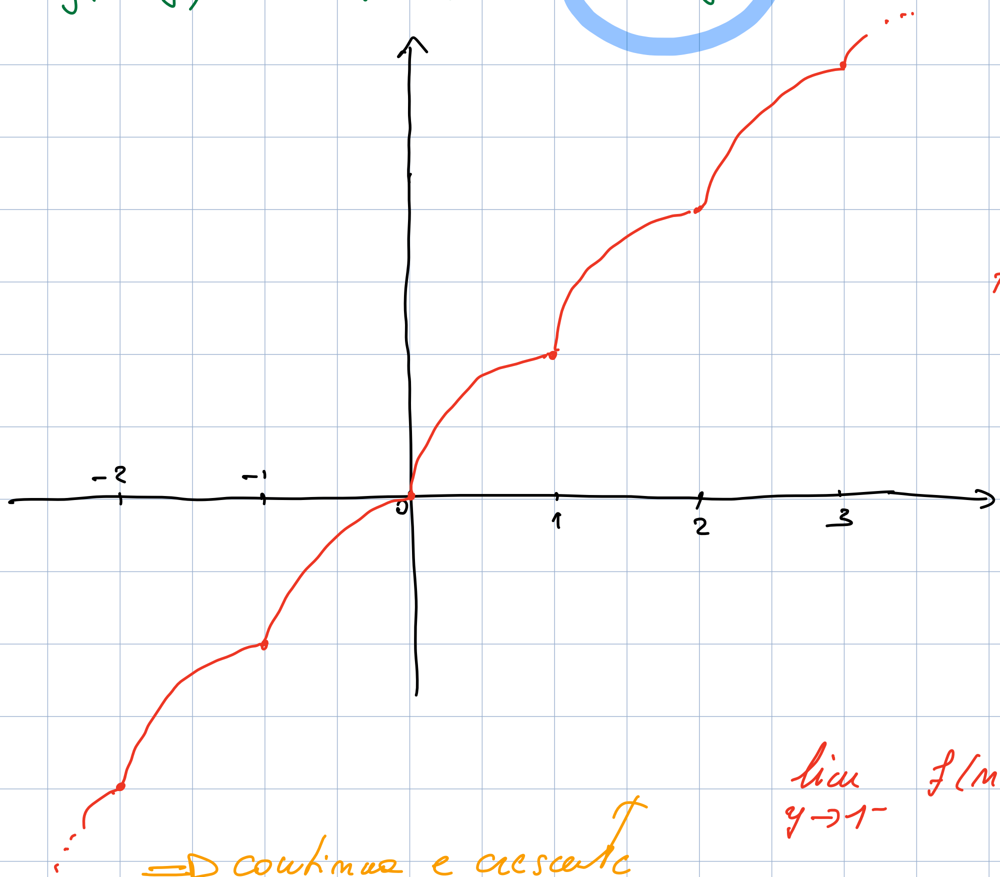
Ist. Mat. I – CIA
16/11/22

Ese Pearson.

(24) $f(x) = \lfloor x \rfloor + \sqrt{x - \lfloor x \rfloor}$ continua e crescente.

$f(n) = n + \sqrt{n-n} = n$

$f(n+y) = n + \sqrt{n+y-n} = n + \sqrt{y}$ $\quad 0 \leq y < 1$



f periodica di periodo 1

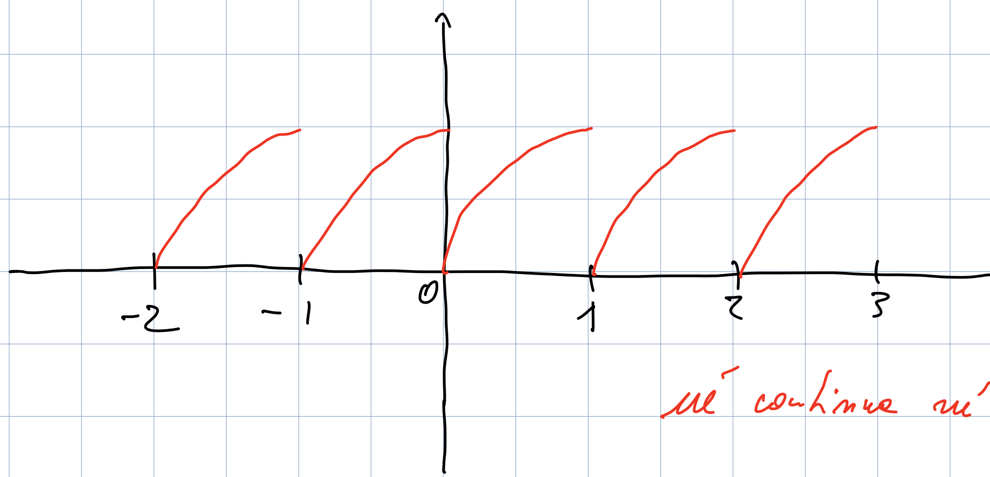
$\lim_{y \to 1^-} f(n+y) = n+1 = f(n+1)$

$\Rightarrow$ continua e crescente

$f(x) = g(x) + h(x)$ $\qquad g(x) = x$ $\qquad$ continua e crescente

$h(x) = \sqrt{x - \lfloor x \rfloor}$

$$h(n+y) = \sqrt{n+y-n} = \sqrt{y}$$



né continua né crescente.

$\boxed{28}$ Disporre in ordine crescente di infinitesimo in $+\infty$ le:

$$f_1(x) = \frac{\sqrt{x}}{x^2 + \cancel{x \cdot \sin(x)} \cancel{-1}} \cong \frac{1}{x^{3/2}}$$

$$f_2(x) = \frac{x^3}{\cosh(x)} = \frac{2x^3}{e^x + e^{-x}} \cong \frac{x^3}{e^x}$$

$$f_3(x) = e^{-x} = \frac{1}{e^x}$$

$$f_4(x) = \frac{x+1}{x^2 \cdot \log(x)} = \frac{1}{x \cdot \log(x)}$$

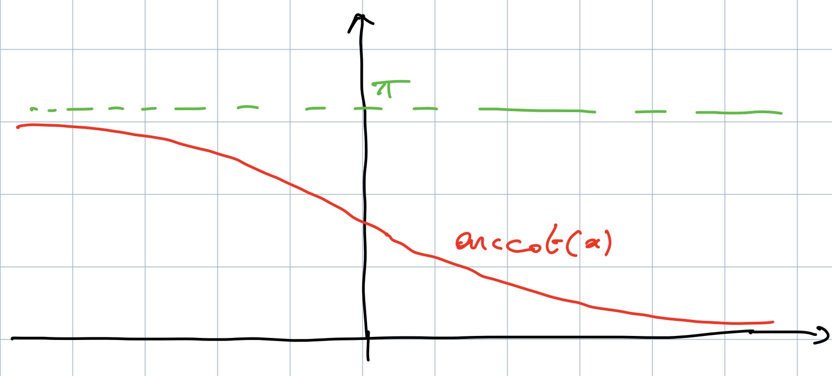
$$f_5(x) = \pi - 2\arctan(x) \cong \frac{1}{x}$$

$$f_6(x) = \left(\frac{1}{\log(x+e^x)}\right)^2 \cong \frac{1}{x^2}$$

$$f_7(x) = x^{-x}$$

$f_5 \quad f_4 \quad f_1 \quad f_6 \quad f_2 \quad f_3 \quad f_7$

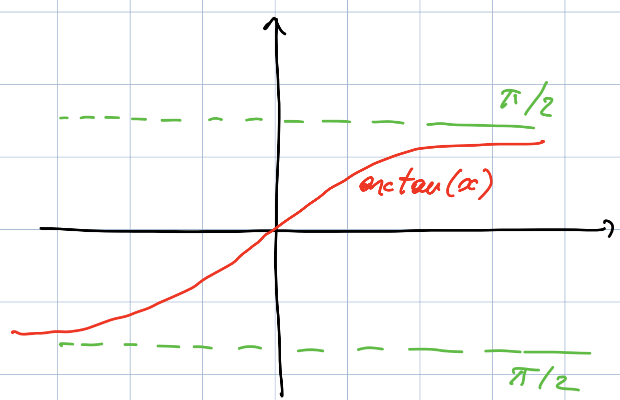
Andamento di $\operatorname{arccot}(x)$ in $+\infty$



$y = \operatorname{arccot}(x)$

so: $\lim_{y \to 0} \frac{y}{\sin(y)} = 1 \implies \lim_{y \to 0} y \cdot \cot(y) = 1$

$\implies \lim_{x \to \infty} x \cdot \operatorname{arccot}(x) = 1 \implies \boxed{\operatorname{arccot}(x) \cong \frac{1}{x} \text{ in } +\infty}$



$\arctan(x) = \frac{\pi}{2} - \operatorname{arccot}(x)$

$\boxed{\frac{\pi}{2} - \arctan(x) \cong \frac{1}{x} \text{ in } +\infty}$

(29) ordine di ∞ in $+\infty$

$$f_1(x) = \log \frac{x}{\cosh(x)-\sinh(x)} = \log \frac{x}{e^{-x}} = \log(x) + x \cong x$$

$$f_2(x) = \sqrt{x}\cdot(2+\sin(x)) \cong \sqrt{x} = x^{1/2}$$

$$f_3(x) = \sqrt{x}\cdot\sqrt{x+1}\cdot\sqrt{x-1} \cong x^{3/2}$$

$$f_4(x) = x^3\cdot\sin\left(\frac{1}{x}\right) \cong x^2$$

f_2 f_1 f_3 f_4

(30) ordine di 0 in 0

$$e^y - 1 \cong y$$
$$e^{-x^2}-1 \cong -x^2$$
$$1-e^{-x^2} \cong x^2$$

$$f_1(x) = 1-e^{-x^2} \cong x^2$$

$$f_2(x) = x^3 + x\cdot\sin(\sqrt{x}) \cong x^3 + x^{3/2} \cong x^{3/2}$$

$$f_3(x) = \sqrt[4]{1+x^3} - \sqrt[3]{1-x^4} \cong 1+\frac{1}{4}x^3 - \left(1+\frac{1}{3}(-x^4)\right) = \frac{1}{4}x^3 + \frac{1}{3}x^4 \cong x^3$$

$$f_4(x) = (1-\cos(x))\log(x) \cong x^2\cdot\log(x)$$

$$f_5(x) = x^{23}$$

$$f_6(x) = \log(1+x)\cdot(1+\log(x)) \cong x\cdot\log(x)$$

$$f_7(x) = 2\arctan(x) \cong x$$

f_6 f_7 f_2 f_4 f_1 f_3 f_5

(31) ordine di ∞ in 0^+

$$f_1(x) = e^{\frac{x^2+1}{x}} = e^{x+\frac{1}{x}} \cong e^{1/x}$$

$$f_2(x) = \log(x^3+x^4) + x \cdot \sin\sqrt{x} \cong \log(x^3) + x^{3/2} \cong \log(x)$$

$$f_3(x) = \frac{1}{2^x-1} \cong \frac{1}{x}$$

$$f_4(x) = \frac{1}{\cosh(x)-\cos(x)} = \frac{2}{e^x+e^{-x}-2\cos(x)}$$

$$= \frac{2}{\underset{\underset{x}{\cong}}{(e^x-1)} + \underset{\underset{x^2}{\cong}}{2(1-\cos(x))} + \underset{\underset{-x}{\cong}}{(e^{-x}-1)}}$$

non posso semplificare questi perché trascuro le componenti asintotiche a x^2 di e^x-1-x
$e^{-x}-1+x$

NON CI SONO AD OGGI GLI STRUMENTI PER DARE ANDAMENTO

Uso $\tilde{f}_4(x) = \frac{1}{x^2}$

$$f_5(x) = \frac{\log(x)}{\sqrt{x^4-x^6}} \cong \frac{\log(x)}{x^2}$$

$$f_6(x) = (\log(e^x-1))^2 \cong (\log(x))^2$$

$f_1(x) \cong e^{1/x}$ $f_2(x) \cong \log(x)$ $f_3(x) \cong \frac{1}{x}$

f_2 f_6 f_3 $\tilde{f}_4$ f_5 f_7

Derivata logaritmica :

$$\boxed{D(\log(f(x))) = \frac{f'(x)}{f(x)}}$$

Teo (Lagrange) : f continua su $[a,b]$ e derivabile su (a,b)
$\Rightarrow \exists c \in (a,b)$ t.c.

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

Teo (Rolle) : stesse ipotesi se $f(b) = f(a)$
$\exists c$ t.c. $f'(c) = 0$.

Teo (Cauchy) : $f, g : [a,b] \to \mathbb{R}$
continue su $[a,b]$ e derivabili in (a,b) con
$g'(x) \neq 0$ $\forall x \in (a,b)$; allora $\exists c$ t.c.

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

Infatti: $h(x) = (f(b)-f(a))\cdot g(x) - (g(b)-g(a))\cdot f(x)$

è continua in $[a,b]$ e derivabile in (a,b); inoltre

$$h(a) = f(b)\cdot g(a) - \cancel{f(a)\cdot g(a)} - g(b)\cdot f(a) + \cancel{g(a)\cdot f(a)}$$

$$h(b) = \cancel{f(b)\cdot g(b)} - f(a)\cdot g(b) - \cancel{g(b)\cdot f(b)} + g(a)\cdot f(b)$$

dunque $h(a) = h(b) \Longrightarrow \exists c$ t.c. $h'(c) = 0$

$$\Longrightarrow \big(f(b)-f(a)\big)\cdot g'(c) = \big(g(b)-g(a)\big)\cdot f'(c).$$ ▨

<u>Fatto</u>: $f:(a,b)\to\mathbb{R}$ derivabile

(1) f crescente $\Longleftrightarrow f'(x)\geq 0 \quad \forall x$

(2) f decrescente $\Longleftrightarrow f'(x)\leq 0 \quad \forall x$

(1) $\underline{\Longrightarrow}$ $f(x+h)-f(x)$ è concorde con h

$$\Longrightarrow \frac{f(x+h)-f(x)}{h}\geq 0$$

$$\Longrightarrow f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\geq 0.$$

$\underline{\Longleftarrow}$ Dati $a < x_1 < x_2 < b$ applico Lagrange a f su $[x_1,x_2]$: $\exists c\in[x_1,x_2]$ t.c.

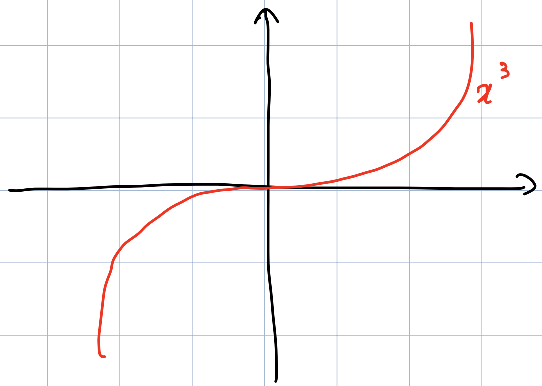
$$f'(c) = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

ma $f'(c) \geq 0$, $x_2 - x_1 > 0$

$\Longrightarrow f(x_2) \geq f(x_1)$ cioè f crescente.

Oss: se $f'(x) > 0$ $\forall x \in (a,b)$ allora f è strettamente crescente (stessa spiegazione)

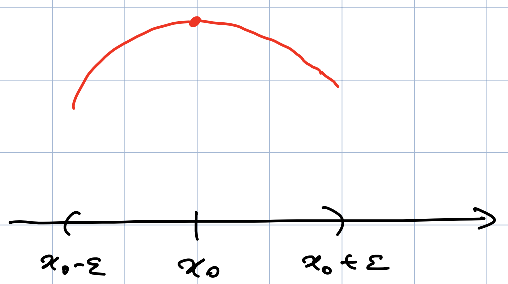
Oss: falso viceversa: $f(x) = x^3$



strett. crescente ma $f'(x) = 3x^2$ si annulla in 0

I metodo di ricerca di max/min locali:

- se $f'(x_0) = 0$, $f'(x) > 0$ per $x \in (x_0 - \varepsilon, x_0)$
  $f'(x) < 0$ per $x \in (x_0, x_0 + \varepsilon)$



$\Longrightarrow x_0$ è punto di max locale

- Se f' passa da $-$ a $+$ attraversando 0 in x_0
  $\Rightarrow$ x_0 pto di min locale

Regola di de l'Hôpital:

$f, g : I \to \mathbb{R}$ $\quad$ I intervallo (qualsiasi tipo)

$x_0 \in \overline{I} = I$ con estremi aggiunti

Inoltre $g(x), g'(x) \neq 0$ $\forall x \in I$.

Supponiamo che

- $\lim\limits_{x \to x_0} \dfrac{f(x)}{g(x)}$ sia $\dfrac{0}{0}$ o $\dfrac{\infty}{\infty}$
- $\lim\limits_{x \to x_0} \dfrac{f'(x)}{g'(x)} = L$

Allora $\lim\limits_{x \to x_0} \dfrac{f(x)}{g(x)} = L$.

Oss 1: ipotesi $g(x), g'(x) \neq 0$ basta vicino a x_0

Oss 2: è essenziale che $\dfrac{f(x)}{g(x)}$ sia $\dfrac{0}{0}$ o $\dfrac{\infty}{\infty}$.

Spiegazione: faccio caso $\dfrac{0}{0}$ in x_0^+ con $x_0 \in \mathbb{R}$.

Per ogni $x > x_0$ posso applicare Cauchy a f, g sull'intervallo $[x_0, x]$: $\quad \exists c \in (x_0, x)$ t.c.

$$\frac{f'(c)}{g'(c)} = \frac{f(x) - \overset{=0}{f(x_0)}}{g(x) - \underset{=0}{g(x_0)}} = \frac{f(x)}{g(x)}$$

$x_0 < c < x$

Se $x \to x_0^+$ anche $c = c(x) \to x_0$

dunque $\frac{f'(c)}{g'(c)} \longrightarrow L$ pertanto $\frac{f(x)}{g(x)} \longrightarrow L$.

Usando la regola provo:

- $\frac{x^\alpha}{e^{\beta x}} \longrightarrow 0$ in $+\infty$ se $\alpha, \beta > 0$

$\alpha = 1$ $\quad \frac{x}{e^{\beta x}} = \frac{\infty}{\infty}$

derivando $\frac{1}{\beta \cdot e^{\beta x}} \longrightarrow 0$

α qualsiasi

$$\frac{x^\alpha}{e^{\beta x}} = \left(\frac{x}{e^{\beta/\alpha \cdot x}}\right)^\alpha \longrightarrow 0$$

- $\frac{(\log_a x)^\alpha}{x^\beta} \longrightarrow 0$ $\quad a > 1$, $\alpha, \beta > 0$

$t = \log_a(x)$ $\quad \frac{t^\alpha}{\gamma^t} \longrightarrow 0$

- $$\lim_{x\to 0} \frac{x-\sin(x)}{x^3} = \frac{1}{6}$$ $\Rightarrow \sin(x) = x - \frac{1}{6}x^3 + o(x^3)$

è forma indeterminata $\frac{0}{0}$

regola: $$\frac{1-\cos(x)}{3x^2} = \frac{1}{3} \cdot \frac{1-\cos(x)}{x^3} \longrightarrow \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6}$$

- $$\lim_{x\to 0} \frac{1-\cos(x)-\frac{1}{2}x^2}{x^4} = -\frac{1}{24}$$ $\Rightarrow \cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)$

è forma indet. $\frac{0}{0}$

regola: $$\frac{\sin(x)-x}{4x^3} = -\frac{1}{4}\cdot\frac{x-\sin(x)}{x^3} \longrightarrow -\frac{1}{4}\cdot\frac{1}{6} = -\frac{1}{24}$$

Esercizio: fare ritorno per e^x, $\cosh(x)$, $\sinh(x)$
+ fa dell'esercizio del Person 21