# Ist. Mat. I - CIA
## 15/12/22

Prove 25/11/22

(3) $\sqrt{3}\cdot z^2 + (\sqrt{2}+i\sqrt{3})z + i\sqrt{2} = 0$

$$\Delta = \underbrace{2+2i\sqrt{6}-3}\ -4i\sqrt{6} = 2-2i\sqrt{6}-3 = (\sqrt{2}-i\sqrt{3})^2 = -1-2i\sqrt{6}$$

$$z_{1,2} = \frac{-\sqrt{2}-i\sqrt{3} \pm (\sqrt{2}-i\sqrt{3})}{2\sqrt{3}} = \begin{cases} -i \\ -\frac{\sqrt{2}}{\sqrt{3}} \end{cases}$$

$$\begin{cases} a^2-b^2=-1 \\ ab=-\sqrt{6} \end{cases} \qquad \begin{array}{l} a=\sqrt{2} \\ b=-\sqrt{3} \end{array}$$

(4) $$\lim_{n\to\infty} \frac{\left[\sqrt{n^2+2n-3}\right]-2n}{n+(-1)^n\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{n-2n}{n} = -1$$

(5) $f:[0,4]\to\mathbb{R}$ $\quad f(x)=\sqrt{x}+\log_3(x+5)$.
$J=\mathrm{Im}(f)$.

$$f = \nearrow + \nearrow = \nearrow \Rightarrow J=[f(0),f(4)] = [\log_3(5),4]$$

Invertibile con inversa continua: ok.

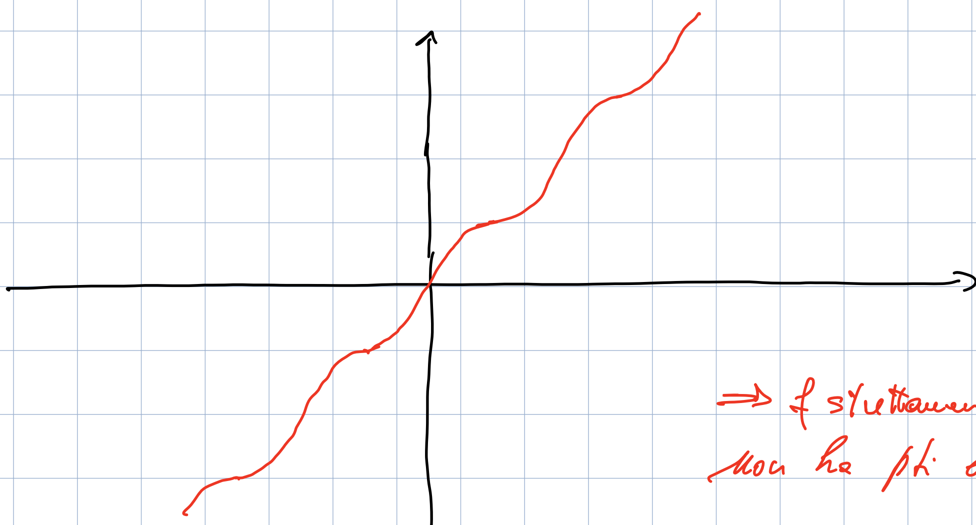
(6) $f : \mathbb{R} \to \mathbb{R} \qquad f(x) = x + \sin(x)$.

$$f'(x) = 1 + \cos(x) \geq 0$$

$$f'(x) = 0 \iff \cos(x) = -1 \iff \pi + 2k\pi, \; k \in \mathbb{Z}.$$

Su ogni $(\pi + 2k\pi, \pi + 2(k+1)\pi)$ ho $f'(x) > 0$

$\Rightarrow$ $f$ è strettamente crescente



$\Rightarrow$ $f$ strettamente crescente
non ha pti estremali.

(7) Taylor IV per $f(x) = e^x \cdot \cos(x)$.

- $f(x) = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + o(x^4)\right) \cdot \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)\right)$

$= 1 + x + \left(-\frac{1}{2} + \frac{1}{6}\right)x^3 + \left(\frac{1}{24} - \frac{1}{4} + \frac{1}{24}\right)x^4 + o(x^4)$

$= \boxed{1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 + o(x^4)}$

- $f(x) = e^x \cdot \cos(x)$ $\quad 1$

$f'(x) = e^x \cdot (\cos(x) - \sin(x))$ $\quad 1$

$f''(x) = e^x \cdot (\cos(x) - \sin(x) - \sin(x) - \cos(x)) = -2e^x \cdot \sin(x)$ $\quad 0$

$f'''(x) = -2e^x \cdot (\sin(x) + \cos(x))$ $\quad -2$

$f^{IV}(x) = -2e^x(\sin(x) + \cos(x) + \cos(x) - \sin(x)) = -4e^x \cdot \cos(x)$

$-4$

$$f(x) = \frac{1}{0!}x^0 + \frac{1}{1!}x^1 + \frac{0}{2!}x^2 + \frac{-2}{3!}x^3 + \frac{-4}{4!}x^4 + o(x^4)$$

$$= \boxed{1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 + o(x^4)}$$

(8) $f : (0, +\infty) \to \mathbb{R}$ $\quad f(x) = \log(x) + x^3$

Trovare $[a, 1]$ su cui si applica il metodo

$$\begin{cases} x_0 = \dots \\ x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)} \end{cases}$$

- $a$ più piccolo possibile
- dire chi è $x_0$.

Voglio che
$f(a) \cdot f(1) < 0$
$f'(x)$ mantiene segno su $[a, 1]$
$f''(x)$ mantiene segno su $[a, 1]$.

$f(1) = 1 > 0$ $\qquad \lim_{x \to 0^+} f(x) = -\infty$

$f'(x) = \frac{1}{x} + 3x^2 > 0$ su $(0, 1]$

$f''(x) = -\frac{1}{x^2} + 6x$ ; $\quad f''(1) = -1 + 6 = 5 > 0$

Voglio $[a,1]$ t.c. $f''(x) > 0$ su $[a,1]$.

$$-\frac{1}{x^2} + 6x > 0 \qquad 6x > \frac{1}{x^2} \qquad 6x^3 > 1$$

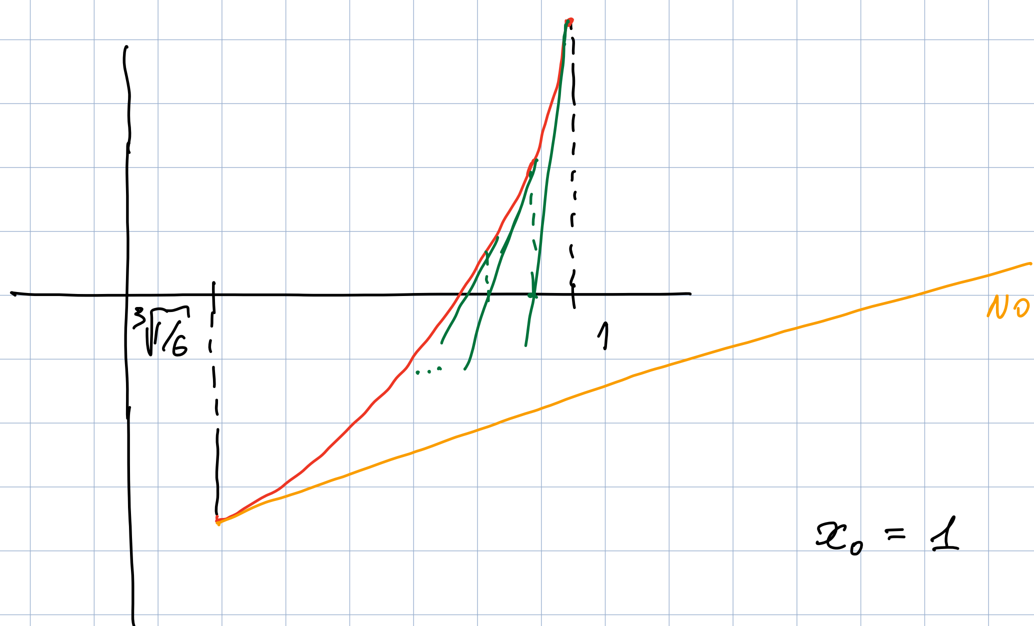
$$x > \sqrt[3]{1/6}$$

Possiamo scegliere $a = \sqrt[3]{1/6}$ perché $f(\sqrt[3]{1/6}) < 0$:

$$\log\left(\sqrt[3]{1/6}\right) + \frac{1}{6} < 0$$

$$-\frac{1}{3}\log(6) + \frac{1}{6} < 0$$

$$2\log(3) > 1 \qquad \underline{\underline{\text{vero}}}.$$




$x_0 = 1$

**Esercizio:** $f(x) = \dfrac{x\cdot(x - |2x-8|)}{x+1}$

Ⓐ $f: D \to \mathbb{R}$

$D = \mathbb{R} \setminus \{-1\}$

$$f(x) = \begin{cases} \dfrac{8x - x^2}{x+1} & x \geq 4 \\[2ex] \dfrac{3x^2 - 8x}{x+1} & x < 4 \end{cases}$$

$$\lim_{x \to -1^\pm} f(x) = \frac{3+8}{0^\pm} = \pm\infty$$

Ⓑ Trovare asintoti:

$x = -1$ verticale

$$\lim_{x \to \pm\infty} f(x) = -\infty$$ no orizzontali.

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{8x - x^2}{x^2 + x} = -1$$

$$\lim_{x \to \infty} \left(f(x) + x\right) = \lim_{x \to \infty} \left(\frac{8x - x^2}{x+1} + x\right) = \lim_{x \to \infty} \frac{8x - x^2 + x^2 + x}{x+1} = 9$$

asintoto obliquo destro $y = 9 - x$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{3x^2 - 8x}{x^2 + x} = 3$$

$$\lim_{x \to \infty} \left(f(x) - 3x\right) = \lim_{x \to \infty} \frac{3x^2 - 8x - 3x^2 - 3x}{x+1} = -11$$

asintoto obliquo sinistro $y = 3x - 11$

Ⓒ Trovare gli zeri di $f$.

$(x \geq 4)$ $\quad x = 8$

$(x < 4)$ $\quad x = 0 \ , \ x = 8/3$

Ⓓ Trovare max/min rel. di $f$.

$(x \geq 4)$

$$f'(x) = \left(\frac{8x - x^2}{x+1}\right)' = \frac{(8-2x)(x+1) - (8x - x^2)}{(x+1)^2}$$

$$= \frac{8x + 8 - 2x^2 - 2x - 8x + x^2}{(x+1)^2} = \frac{8 - 2x - x^2}{(x+1)^2}$$

$f'(x) = 0$ per $x^2 + 2x - 8 = 0$ $\quad (x+4)(x-2)$

$x = -4, x = 2$ fuori da $[+4, +\infty)$

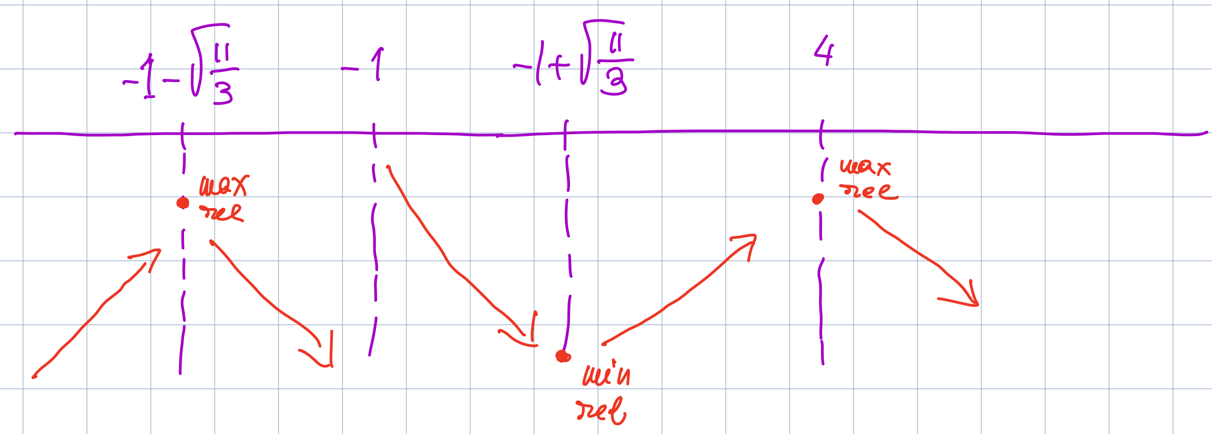
$\Rightarrow f'(x) < 0$ su $[+4, +\infty)$

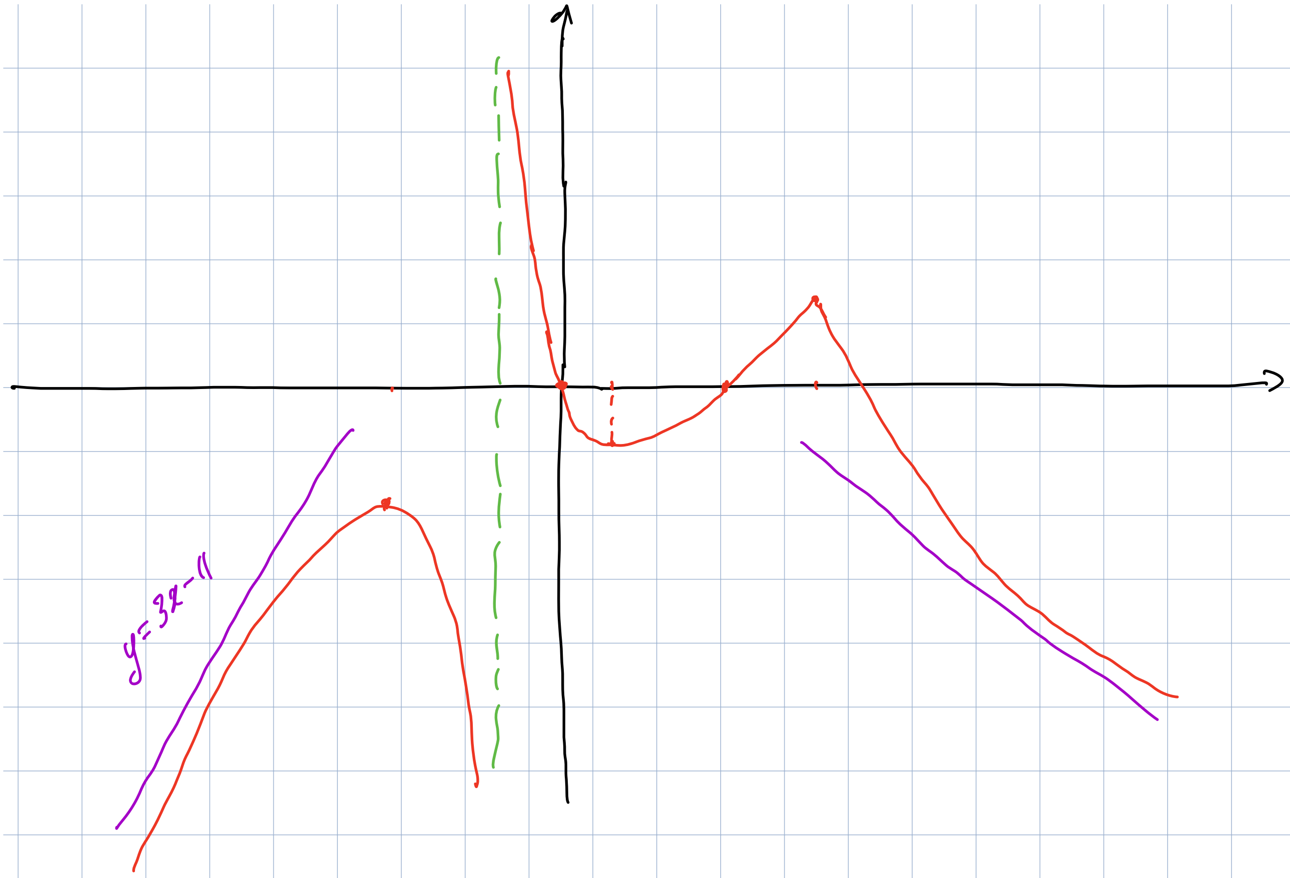
$(x < 4)$

$$f'(x) = \left(\frac{3x^2 - 8x}{x+1}\right)' = \frac{(6x-8)(x+1) - (3x^2 - 8x)}{(x+1)^2}$$

$$= \frac{6x^2 + 6x - 8x - 8 - 3x^2 + 8x}{(x+1)^2} = \frac{3x^2 + 6x - 8}{(x+1)^2}$$

Nulla in $\frac{-3 \pm \sqrt{9+24}}{3} = -1 \pm \sqrt{\frac{11}{3}}$






(esercizio: studiare conc/conv.)

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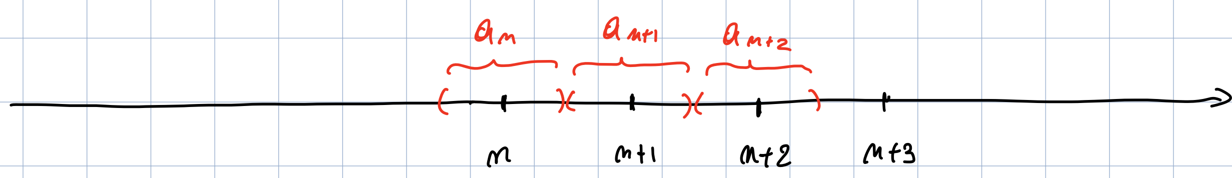
Prova del 15/12/22

① $a_n = n + (-1)^n \frac{7}{n}$ monotona per $n$ grande?

$1-7=-6$, $2+\frac{7}{2}=5.5$, $3-\frac{7}{3}=0.66\ldots$, $4+\frac{7}{4}=5.\ldots$

Per $n$ grande $a_n \cong n$ : crescente.

Per $n \geq 15$ l'intero più vicino ad $a_n$ è $n$




(2) Quante combinaz. di segni diversi si possono ottenere estraendo 5 carte da mazzo da 40.

| Numeri di carte per segno | Combinazioni possibili | |
|---|---|---|
| $5+0+0+0$ | $4$ | |
| $4+1+0+0$ | $4\cdot 3 = 12$ | |
| $3+2+0+0$ | $4\cdot 3 = 12$ | |
| $3+1+1+0$ | $4\cdot\binom{3}{2} = 4\cdot 3 = 12$ | |
| $2+2+1+0$ | $\binom{4}{2}\cdot 2 = \frac{4\cdot 3}{2}\cdot 2 = 12$ | $4\cdot 3 = 12$ |
| $2+1+1+1$ | $4$ | |

$$\Longrightarrow 12\times 4 + 4\times 2 = 56$$

(3) $$\lim_{x\to 0} \frac{\log(1+\tan^2(x))}{\cos(x)-1}$$

$$\frac{\log(1+\tan^2(x))}{\cos(x)-1} \simeq \frac{\tan^2(x)}{\left(1-\frac{1}{2}x^2\right)-1} \simeq \frac{x^2}{-\frac{1}{2}x^2} = -2$$

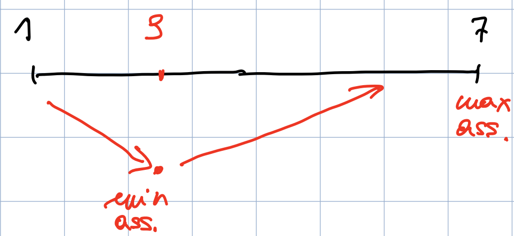
④ $f: [1,7] \to \mathbb{R}$ $\quad f(x) = x - 4 \cdot \log(1+x)$.

Provare che ha max/min ass. e trovare i pti ...

f è continua su chiuso e lim.

f è derivabile $\Rightarrow$ i pti sono gli estremi o quelli staz.

$$f'(x) = 1 - \frac{4}{1+x} = \frac{x-3}{1+x}$$




$f(1) = 1 - 4\log(2)$ $\qquad f(7) = 7 - 4\log(8) = 7 - 12\log(2)$

$$1 - 4\log(2) < 7 - 12\log(2)$$

$\iff \quad 8\log(2) < 6$

$\iff \quad 4\log(2) < 3$

$\iff \quad 16 < e^3$ $\qquad e = 2{,}71\ldots$ $\qquad$ SI

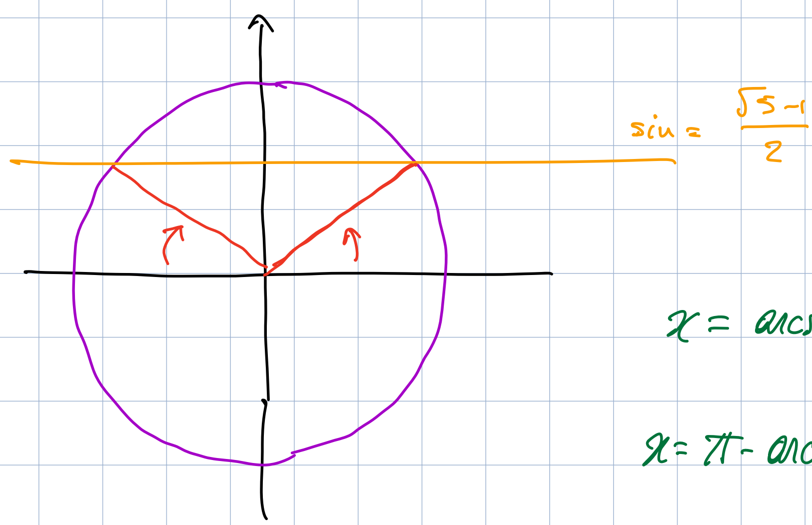
⑤ $f(x) = \cos(x)\cdot e^{\sin(x)}$ ; trovare $f'(x)$ dove esiste e dove si annulla.

$f'(x)$ esiste sempre

$$\begin{aligned} f'(x) &= -\sin(x)\cdot e^{\sin(x)} + \cos^2(x)\cdot e^{\sin(x)} \\ &= e^{\sin(x)}\cdot\left(\cos^2(x) - \sin(x)\right) \\ &= e^{\sin(x)}\cdot\left(1 - \sin(x) - \sin^2(x)\right) \end{aligned}$$

$$t^2+t-1=0 \qquad t_{1,2}=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

$$f'(x)=0 \iff \sin(x)=\frac{\sqrt{5}-1}{2}$$




$$x=\arcsin\left(\frac{\sqrt{5}-1}{2}\right)+2k\pi$$

$$x=\pi-\arcsin\left(\frac{\sqrt{5}-1}{2}\right)+2k\pi$$

⑥ $f(x)=1+\operatorname{sgn}(x)\cdot e^{-\frac{1}{x}}-x^2$

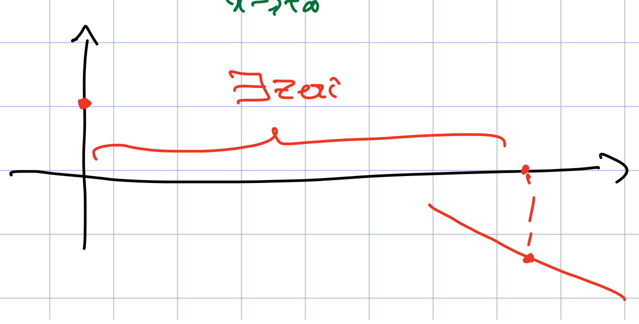
$\lim_{x\to 0^+} f(x)=1$ su $[0,+\infty)$

$\lim_{x\to 0^-} f(x)=+\infty$ su $(-\infty,0)$

Proviamo che ci sono intervalli su cui si applica teo zeri.

Su $[0,+\infty)$

$f(0)=1 \qquad \lim_{x\to+\infty} f(x)=1+1-\infty=-\infty$



⑦ $\sum \left(1+\frac{1}{n}\right)^{n^2}$

$\sqrt[n]{a_n} = \left(1+\frac{1}{n}\right)^n \longrightarrow e > 1$ diverge