Ist. Mat. I — CIA
13/12/23

$\boxed{10/1/23}$ $\quad x\cdot\log(x^2)$

(A) Provare che esso si estende a $f:\mathbb{R}\to\mathbb{R}$ continua

$x\cdot\log(x^2)$ def. per $x^2>0$ cioè $x\neq 0$.

$$\lim_{x\to 0}\left(x\cdot\log(x^2)\right) = 0\cdot(-\infty)$$
$$= 0$$

$$f(x) = \begin{cases} x\cdot\log(x^2) & x\neq 0 \\ 0 & x=0. \end{cases}$$

(B) Trovare zeri di f.

0 ; $\quad x\cdot\log(x^2)=0 \Rightarrow \log(x^2)=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$

(Oss: f è dispari.)

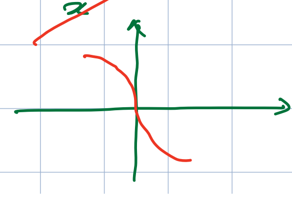
(C) Dire dove esiste f' e tipo punto dove non esiste.

- f' esiste sempre per $x\neq 0$

$\exists f'(0)$? $\quad \lim_{x\to 0}\dfrac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\dfrac{\cancel{x}\cdot\log(x^2)-0}{\cancel{x}} = -\infty$

$\Rightarrow f'(0)$ non esiste.

o flesso a tg. verticale.



- per $x \neq 0$ $\quad f'(x) = \left(x \cdot \log(x^2)\right)'$

$$= \log(x^2) + x \cdot \frac{1}{x^2} \cdot 2x = \log(x^2) + 2$$

$\Rightarrow f'(x)$ esiste $\forall x \neq 0$

$\lim_{x \to 0} f'(x) = -\infty \Rightarrow \dots$

($\underline{\text{Oss}}$: $\quad f(x) = x \cdot \log(x^2) = 2x \cdot \log(|x|)$

$\Rightarrow f'(x) = 2 \cdot \log|x| + \frac{2x}{x} = \dots$)

(D) Trovare inf. e sup.

$x \cdot \log(x^2)$ $\qquad \lim_{x \to \pm\infty} f(x) = \pm\infty$

$\inf = -\infty \quad , \quad \sup = +\infty$

(E) Trovare asintoti.

vert. $\underline{\text{no}}$; orizz. $\underline{\text{no}}$;

obl? $\quad \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \log(x^2) = +\infty \qquad \underline{\text{No}}$

(F) Trovare max / min rel.

$f'(x) = \log(x^2) + 2$

$$f'(x) = 0 \iff 2\log(|x|) = -2$$

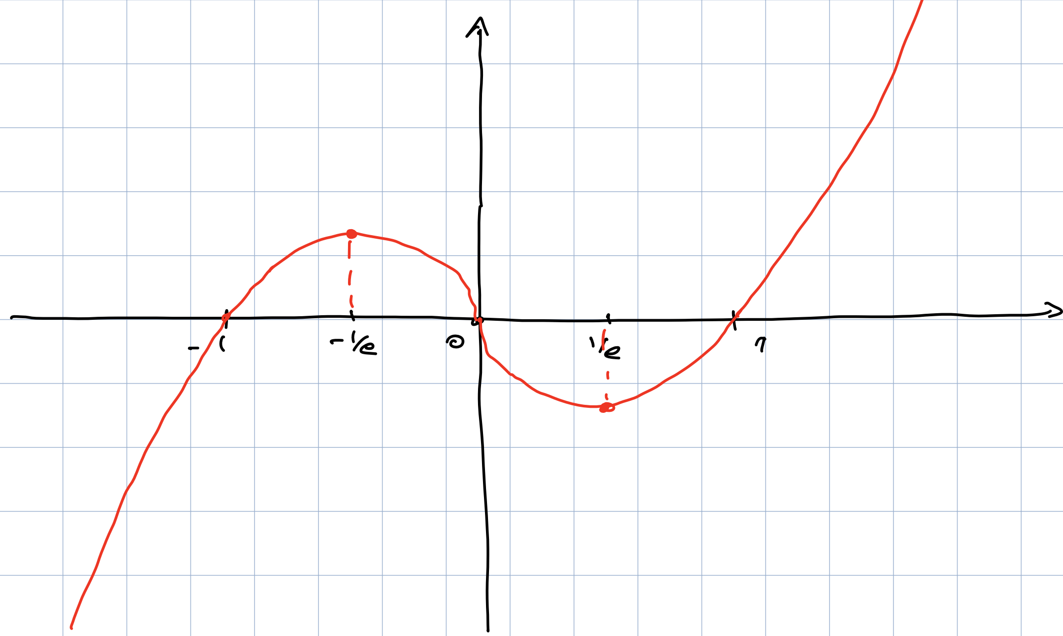
$$\log(|x|) = -1$$

$$|x| = \frac{1}{e}$$

$$x = \pm\frac{1}{e}$$

		$-1/e$		0		$1/e$	
f'	$+$	0	$-$	$-\infty$	$-$	0	$+$
		max rel.				min rel.	

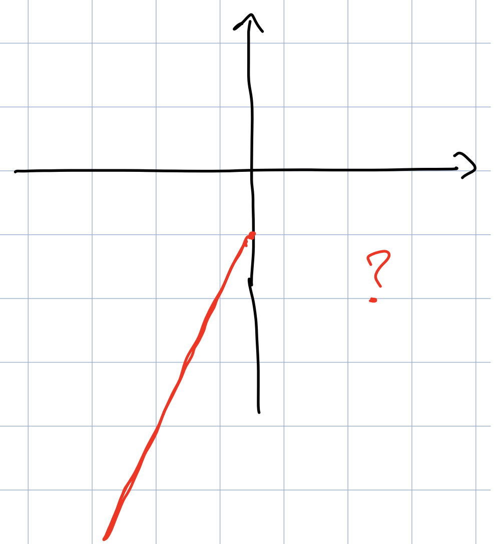
$$f\left(\pm\frac{1}{e}\right) = \pm\frac{1}{e}\cdot(-2) = \mp\frac{2}{e}$$



$\boxed{31/1/23}$

$$\frac{4x^2+4|x|+1}{2x-1} = \begin{cases} \dfrac{4x^2+4x+1}{2x-1} & x \geq 0 \\[2ex] \dfrac{4x^2-4x+1}{2x-1} = \dfrac{(2x-1)^2}{2x-1} = 2x-1 & x<0 \end{cases}$$

(A) Trovare D più grande possibile t.c. una definire una $f: D \to \mathbb{R}$.



$D = \mathbb{R} \setminus \{1/2\}$

(B) Asintoti:

$y = 2x-1$ obl. sin.

$$\lim_{x \to \left(\frac{1}{2}\right)^{\pm}} f(x) = \frac{4 \cdot \frac{1}{4} + 4 \cdot \frac{1}{2} + 1}{0^{\pm}} = \pm\infty \quad ; \quad x = \frac{1}{2} \text{ vert.}$$

$$\lim_{x \to +\infty} f(x) = +\infty \quad \text{no orizz.}$$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim \frac{4x^2 + \cdots}{2x^2 + \cdots} = 2 = m$$

$$f(x) - m \cdot x = \frac{4x^2+4x+1}{2x-1} - 2x = \frac{\cancel{4x^2}+4x+1\cancel{-4x^2}+2x}{2x-1} \to 3$$

$y = 2x+3$ obl.

(C) Trovare punti in cui $\nexists f'$ e dire tipo.

- $f'(x)$ esiste di certo per $x \neq 0, 1/2$

$$\exists f'(0) = \lim_{x \to 0^\pm} \dots$$

- $x>0$: $f'(x) = \left(\dfrac{4x^2+4x+1}{2x-1}\right)' =$

$$= \frac{(8x+4)(2x-1) - 2(4x^2+4x+1)}{(2x-1)^2}$$

$$= \frac{16x^2 \cancel{-8x} \cancel{+8x} - 4 - 8x^2 - 8x - 2}{(2x-1)^2}$$

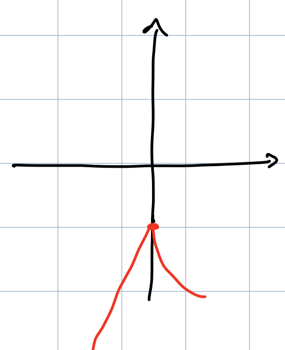
$$= \frac{2 \cdot (4x^2-4x-3)}{(2x-1)^2} \xrightarrow[x \to 0^+]{} \frac{-6}{1} = -6$$

$$\Rightarrow f'_+(0) = -6$$

$x<0$ $\quad f'(x) = 2 \Longrightarrow f'_-(0) = 2$

$\nexists f'(0)$; 0 punto angoloso.

(D) Trovare max/min. rel.



0 max. rel.

zeri di $f'(x)$ per $x>0$:

$f'(x)$ concorde con $4x^2-4x-3$

nulle in $\frac{2 \pm \sqrt{4+12}}{4} = \frac{2 \pm 4}{4} = \begin{cases} 3/2 \\ -1/2 \end{cases}$

$$f' \quad \begin{array}{ccccc} 0 & & 3/2 & & \\ \vdash & - & \text{min rel.} & + & \end{array}$$

(E) Trovare intervalli su cui f è str. conc/conv.

$x > 0$; $\frac{f''(x)}{2} = \left(\frac{4x^2 - 4x - 3}{(2x-1)^2} \right)'$

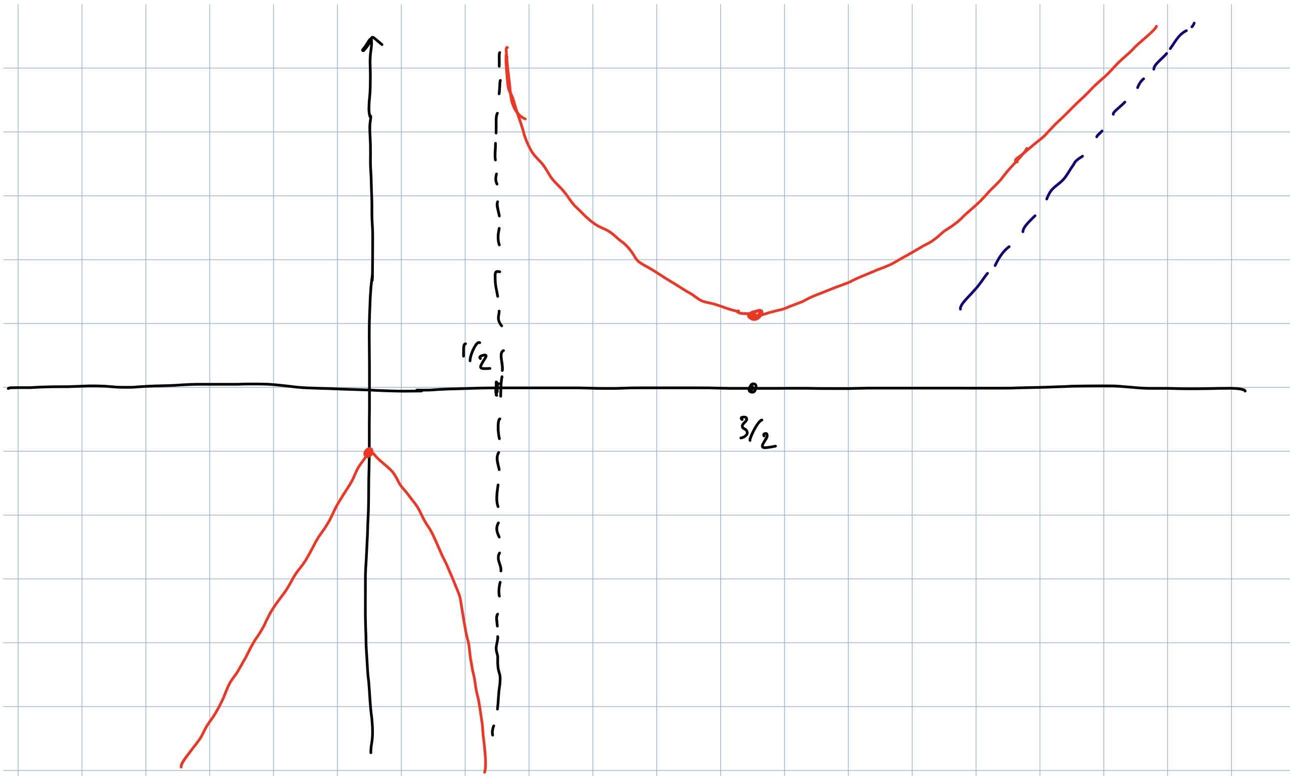
$$= \frac{(8x-4)\cdot(2x-1)^{\cancel{2}\,1} - 2\cdot\cancel{(2x-1)}\cdot 2\cdot(4x^2-4x-3)}{(2x-1)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{16x^2} \cancel{- 8x} \cancel{- 8x} + 4 \cancel{- 16x^2} \cancel{+16x} + 12}{(2x-1)^3} = \frac{16}{(2x-1)^3}$$

concorde con $2x-1$

$$f'' \quad \begin{array}{ccccccc} & 0 & & 1/2 & & \\ 0 & \times & - & 0 & + \end{array}$$

str. concava su $[0, 1/2)$; str. conv. su $(1/2, +\infty)$



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$$f(x) = \sqrt[3]{x^3 - 2x^2 + 1}$$

(A) Trovare D t.c. $f: D \to \mathbb{R}$.

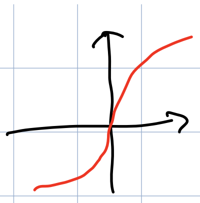
$$D = \mathbb{R}$$

(B) limiti agli estremi.

$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

(C) Trovare dove non esiste f' dicendo tipo.

Ricordo: $\sqrt[3]{y}$ è derivabile per $y \neq 0$.



$\Rightarrow f$ derivabile dove $x^3-2x^2+1 \neq 0$.

$x^3-2x^2+1 = (x-1)(x^2-x-1)$

nullo per $x=1$ e $x = \frac{1 \pm \sqrt{5}}{2}$ $\frac{1+\sqrt{5}}{2} > \frac{3}{2}$, $\frac{1-\sqrt{5}}{2} < -\frac{1}{2}$

Negli altri punti:

$$f'(x) = \frac{1}{3} \underbrace{(x^3-2x^2+1)^{-\frac{2}{3}}}_{\downarrow \, +\infty} \cdot (3x^2-4x)$$

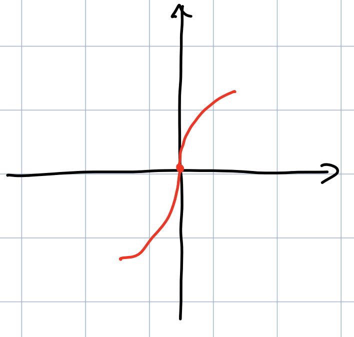
	$\downarrow$	
$x=1$	-1	$-\infty$
$x=\frac{1+\sqrt{5}}{2}$	>0	$+\infty$
$\frac{1-\sqrt{5}}{2}$ ...	>0	$+\infty$

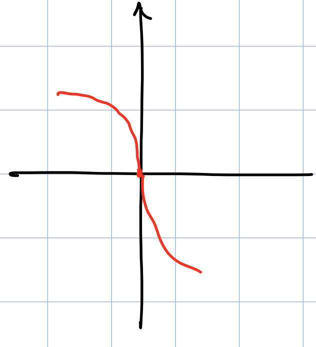
flessi a tangente verticale

$g : (a,b) \to \mathbb{R}$ $t \in (a,b)$

$\exists g'(x) \ \forall x \neq t$

$\lim_{x \to t} g'(x) = \pm\infty$

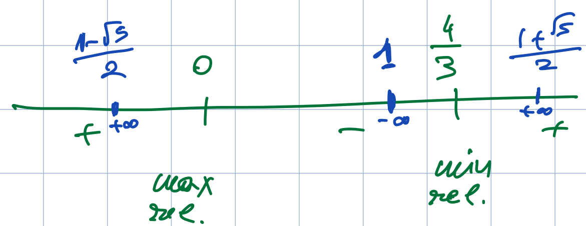




(D) Trovare max/min rel.

I punti di non derivabilità non lo sono.

$f'(x)$ concorde con $3x^2-4x = x(3x-4)$



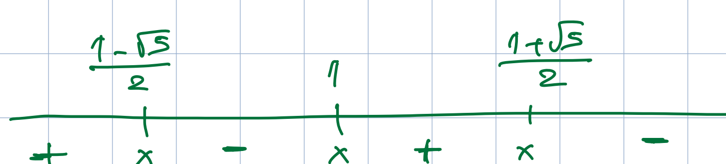
(E) Intervalli di concavità convessità

$$3f''(x) = \left(\left(x^3-2x^2+1\right)^{-\frac{2}{3}}\cdot\left(3x^2-4x\right)\right)'$$

$$= -\frac{2}{3}\left(x^3-2x^2+1\right)^{-\frac{5}{3}}\cdot\left(3x^2-4x\right)^2 + \left(x^3-2x^2+1\right)^{-\frac{2}{3}}\cdot(6x-4)$$

$$= \frac{2}{3}\left(x^3-2x^2+1\right)^{-\frac{5}{3}}\cdot\left(-\left(3x^2-4x\right)^2 + 3\cdot\left(x^3-2x^2+1\right)\cdot(3x-2)\right)$$

$$= \underbrace{-\frac{2}{3}\left(x^3-2x^2+1\right)^{-\frac{5}{3}}}_{\text{discorde da } x^3-2x^2+1}\cdot\underbrace{\left(4x^2-9x+6\right)}_{\substack{\Delta = 81-96<0 \\ \text{sempre positivo}}}$$



s/n. Convessa in $\left(-\infty, \frac{1-\sqrt{5}}{2}\right]$ e $\left[1, \frac{1+\sqrt{5}}{2}\right]$

s/n. concava in $\left[\frac{1-\sqrt{5}}{2}, 1\right]$ e $\left[\frac{1+\sqrt{5}}{2}, +\infty\right)$

(F) Trovare asintoti.

No vert. No orizz.

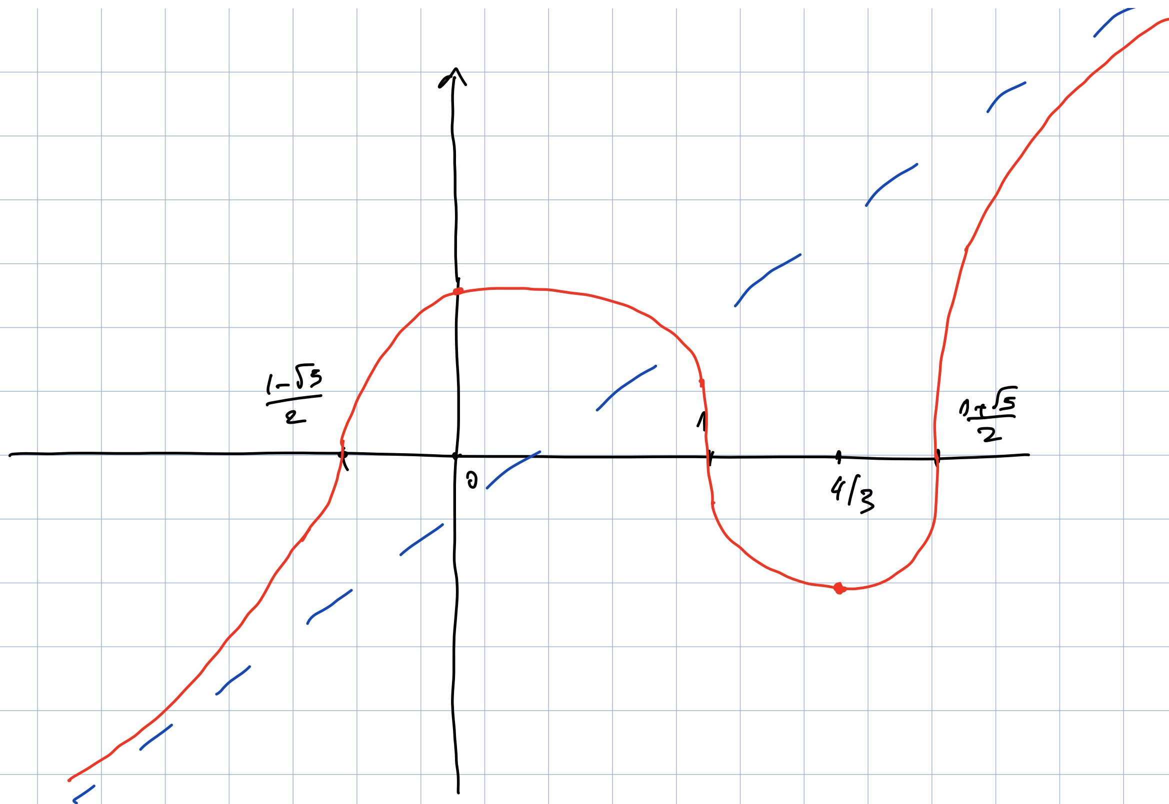
Obl? $\lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim \frac{\sqrt[3]{x^3+\dots}}{x} = 1 = m(?)$

q? $f(x)-x = \sqrt[3]{x^3-2x^2+1} - x$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$$f(x)-x = \frac{\cancel{x^3}-2x^2+1-\cancel{x^3}}{\underbrace{\underbrace{\Big(\underbrace{\sqrt[3]{\dots}}_{x}\Big)^2}_{x^2}+\underbrace{\underbrace{\sqrt{\dots}}_{x}\cdot x}_{x^2}+\underbrace{x^2}_{x^2}}_{3x^2}} \longrightarrow -\frac{2}{3}$$

$y = x - \frac{2}{3}$ asintoto obl.



$$f(x) = \frac{3^x - 1}{3^{2x} - 9} = \frac{3^x - 1}{(3^x)^2 - 3^2} = \frac{3^x - 1}{(3^x - 3)\cdot(3^x + 3)}$$

(A) D $\qquad 3^{2x} - 9 = 0 \iff x = 1$

$$D = \mathbb{R} \setminus \{1\}$$

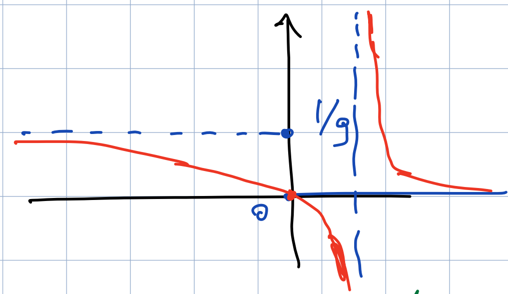
(B) Zeri $\qquad 3^x = 1 \iff x = 0$

(C) limiti agli estremi

$$\lim_{x \to -\infty} \frac{3^x - 1}{3^{2x} - 9} = \frac{1}{9}$$ asintoto orizz. sinistro.

sopra o sotto vicino a $-\infty$?

$$\frac{3^x-1}{3^{2x}-9}-\frac{1}{9}=\frac{9\cdot 3^x-9-3^{2x}+9}{3^{2x}-9}=\frac{9\cdot 3^x-3^{2x}}{3^{2x}-9}\to\frac{0^+}{-9}=0^-$$



$$\lim_{x\to 1^\pm}\frac{3^x-1}{3^{2x}-9}=\frac{2}{0^\pm}=\pm\infty$$

$x=1$ asintoto vert.

$$\lim_{x\to+\infty}\frac{3^x-1}{3^{2x}-9}=0^+$$

(D) Provare che $f'(x)<0\ \forall x$

$$\left(\frac{3^x-1}{3^{2x}-9}\right)'=\frac{3^x\cdot\log(3)\cdot(3^{2x}-9)-3^{2x}\cdot\log(3)\cdot 2\cdot(3^x-1)}{(3^{2x}-9)^2}$$

$$=\frac{3^x\cdot\log(3)}{(3^{2x}-9)^2}\cdot\left(3^{2x}-9-2\cdot 3^{2x}+2\cdot 3^x\right)$$

$$=-\underbrace{\frac{3^x\cdot\log(3)}{(3^{2x}-9)^2}}_{>0}\cdot\Big(\underbrace{3^{2x}-2\cdot 3^x+9}_{t=3^x}\Big)$$

t^2-2t+9 $\qquad \Delta/4=1-9<0$

> 0

< 0

(E) f decr.

No (uso che $f' < 0 \Rightarrow f \searrow$ solo su intervallo)