Calcolo integrale

1. $\sqrt{4x^2+4x+2} = 2x+t = \dots = \dfrac{-t^2+2t-2}{2(1-t)}$

$x = \dfrac{t^2-2}{4(1-t)}$

$dx = \dfrac{-t^2+2t-2}{4(1-t)^2}\,dt$

$$y = \int \frac{\frac{t^2-2}{2(1-t)}}{\frac{-t^2+2t-2}{2(1-t)}} \; \frac{-t^2+2t-2}{4(1-t)^2}\,dt = \int \frac{t^2-2}{4(t-1)^2}\,dt =$$

$$= \frac{1}{4}\int\left\{1+\frac{2t-3}{(t-1)^2}\right\}dt = \frac{1}{4}\int\left\{1+\frac{2}{t-1}+\left(\frac{1}{t-1}\right)'\right\}dt =$$

$$= \frac{1}{4}t + \frac{1}{2}\lg|t-1| + \frac{1}{4(t-1)} + c$$

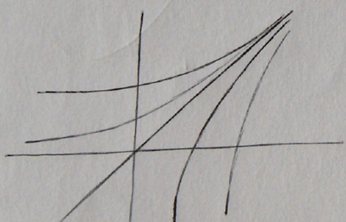
$t = \sqrt{4x^2+4x+2} - 2x$

2. Non esistono soluzioni costanti.
Separando le variabili e integrando:

$$\int_{y_0}^{y} e^s\,ds = \int_{x_0}^{x} e^s\,ds \Rightarrow e^y = e^x + c \Rightarrow y = \lg(e^x + c)$$

Deve essere:

$e^x + c > 0 \Rightarrow$ se $c \geq 0$, $x \in \mathbb{R}$
se $c < 0$, $x > \lg(-c)$



3. $\sqrt[n]{|a_n|} = \dfrac{2n-1}{n+1}|x| \to 2|x|$

Se $|x| < \frac{1}{2}$, la serie conv. (ass.)
Se $|x| > \frac{1}{2}$, la serie non conv.

Se $x = \frac{1}{2}$, $a_n = \left(\dfrac{2n-1}{2n+2}\right)^n$

Se $x = -\frac{1}{2}$, $a_n = (-1)^n\left(\dfrac{2n-1}{2n+2}\right)^n$

$$\left(\frac{2n-1}{2n+2}\right)^n = e^{n\lg\frac{2n-1}{2n+2}} \sim e^{n\left(\frac{2n-1}{2n+2}-1\right)} \to e^{-3/2}$$

In questi due casi, non essendo verificata la condizione necessaria, la serie non converge.

4.

$$\left.\begin{array}{ll} \text{Per } t \to 0 & f(t) \sim -\dfrac{1}{t} \\ \text{per } t \to 1 & f(t) \sim \dfrac{e}{t-1} \end{array}\right\} \text{ infinito di ordine 1}$$

per $t \to +\infty$ $\quad f(t) \sim \dfrac{e^t}{t^2} \to +\infty$ non è verificata la condizione necessaria

per $t \to -\infty$ $\quad f(t) \sim \dfrac{e^t}{t^2} < \dfrac{1}{t^2}$ infinitesimo di ordine 2

Studio di $F(x)$

C.E. $\quad (-\infty, 1/2) \cup (1, +\infty)$

SGN $\quad F(x) > 0$ per $x \in (1, +\infty)$, $F(x) < 0$ per $x \in (-\infty, 1/2)$

LIM $\quad$ per $x \to 0$ $\quad F(x) \sim \int_x^{2x} -\frac{1}{t}\, dt \to -\lg 2$ (discont. eliminabile)

per $x \to \frac{1}{2}^-$ $\quad F(x) \to \int_{\frac{1}{2}}^{1} f(t)\, dt = -\infty$ (asintoto verticale)

per $x \to 1^+$ $\quad F(x) \to \int_1^2 f(t)\, dt = +\infty$ (asintoto verticale)

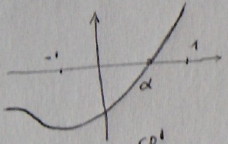
per $x \to -\infty$ $\quad F(x) \to 0$ (asintoto orizzontale)

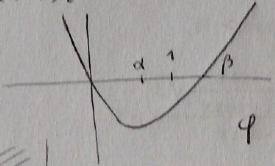
per $x \to +\infty$ $\quad F(x) = \dfrac{x e^{\xi}}{\xi(\xi-1)} \sim \dfrac{x e^{\xi}}{\xi^2} > \dfrac{x e^x}{4x^2} \to +\infty$ senza asint.

DRV $\quad F'(x) = \dfrac{1}{x}\left(\dfrac{e^{2x}}{2x-1} - \dfrac{e^x}{x-1}\right) = \dfrac{e^x}{x(2x-1)(x-1)}\left(e^x(x-1) - 2x + 1\right)$

Studio della funzione $\varphi(x) = e^x(x-1) - 2x + 1$

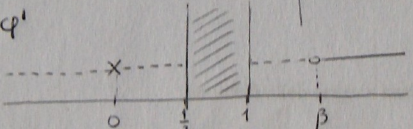
$\varphi'(x) = x e^x - 2 \qquad \varphi''(x) = (x+1)e^x$

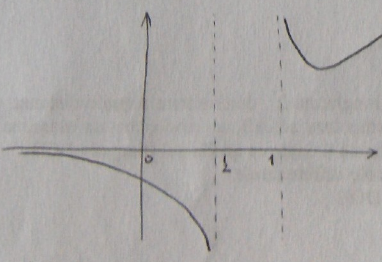




φ' $\qquad\qquad$ φ

segno F'





per $x \to 0$

$F'(x) \sim \dfrac{e^x(x-1) - 2x + 1}{x} \to -2$