Soluzioni [1]

1. $$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \implies \cos x^2 = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$$

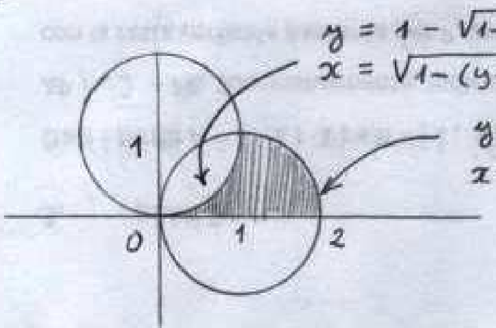
$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{x^4}{(2n+1)(2n+2)} \to 0 \text{ ; la serie converge } \forall x \in \mathbb{R}$$

$$\int_0^1 \cos x^2 \, dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \int_0^1 x^{4n} dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!(4n+1)}$$

Per l'osservazione al teorema di Leibniz sulle serie a segno alterno:

$$|E_n| < \frac{1}{(2n+2)!\,(4n+5)} < 10^{-3} \quad \text{per } n \geq 2$$

$$\int_0^1 \cos x^2 \, dx \sim 1 - \frac{1}{10} + \frac{1}{120} \sim 0.908 \quad \text{appross. per eccesso.}$$

2. 

Metodo gusci cilindrici per il vol.

$$V = 2\pi \int_0^1 x\left(1-\sqrt{1-x^2}\right) dx + 2\pi \int_1^2 x\sqrt{1-(x-1)^2}\, dx \quad \leftarrow x-1=t$$

$$= 2\pi \int_0^1 x\, dx - \cancel{2\pi \int_0^1 x\sqrt{1-x^2}\, dx} + 2\pi \int_0^1 \sqrt{1-t^2}\, dt + \cancel{2\pi \int_0^1 t\sqrt{1-t^2}\, dt}$$

$$= \pi + 2\frac{\pi^2}{4} = \pi\left(1+\frac{\pi}{2}\right).$$

[Area
Metodo sezioni ↓

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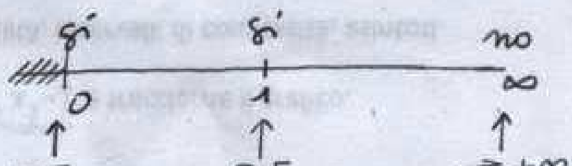
3. $$f(t) = \frac{t(t-1)}{\lg t}$$



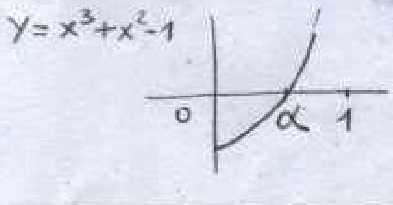
$F(x)$ C.E. $x \geq 0$ LIM per $x \to +\infty$ $F(x) = \frac{(x^2-x)\,\xi(\xi-1)}{\lg \xi}$

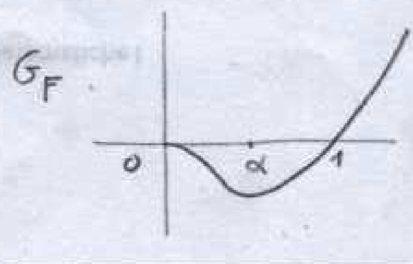
SGN: 0 $-$ 1 $+$

$$\sim \frac{x^2 \xi^2}{\lg \xi} > \frac{x^4}{2\lg x} \to +\infty$$

senza asintoto

DRV $F'(x) = \frac{x(x-1)}{\lg x}(x^3+x^2-1)$





$F'(0) = 0$
$F'(1) = 1$

2. cont.

Area:

$$\cdot\ A = \int_0^1 (1-\sqrt{1-x^2})\,dx + \frac{\pi}{4} = 1$$

come si può dedurre facilmente per via geometrica

Volume: metodo sezioni

$$V = \pi \int_0^1 \left[(1+\sqrt{1-y^2})^2 - (1-(y-1)^2 \right] dy$$

$$= \pi \int_0^1 (2-2y+2\sqrt{1-y^2})\,dy = \pi\left(1+\frac{\pi}{2}\right)$$

4. Soluzioni eq. omogenea $y_0(x) = c_1 \cos 2x + c_2 \operatorname{sen} 2x$

Soluzione particolare:

$$W(x) = \begin{pmatrix} \cos 2x & \operatorname{sen} 2x \\ -2\operatorname{sen} 2x & 2\cos 2x \end{pmatrix}$$

$$c_1' = \det \begin{pmatrix} 0 & \operatorname{sen} 2x \\ 1/\operatorname{sen} x & 2\cos 2x \end{pmatrix} \Big/ \det W(x) = -\cos x$$

$$c_2' = \det \begin{pmatrix} \cos 2x & 0 \\ -2\operatorname{sen} 2x & 1/\operatorname{sen} x \end{pmatrix} \Big/ \det W(x) = \frac{1}{2\operatorname{sen} x} - \operatorname{sen} x$$

$$c_1 = -\operatorname{sen} x, \quad c_2 = \frac{1}{2}\lg\left|\operatorname{tg}\frac{x}{2}\right| + \cos x$$

$$\bar{y}(x) = -\operatorname{sen} x \cos 2x + \cos x \operatorname{sen} 2x + \frac{1}{2}\operatorname{sen} 2x \lg\left|\operatorname{tg}\frac{x}{2}\right|$$

1. $\operatorname{sen} x = \sum_{m=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \Rightarrow \operatorname{sen} x^2 = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{x^4}{(2n+2)(2n+3)} \to 0$; la serie converge $\forall x \in \mathbb{R}$

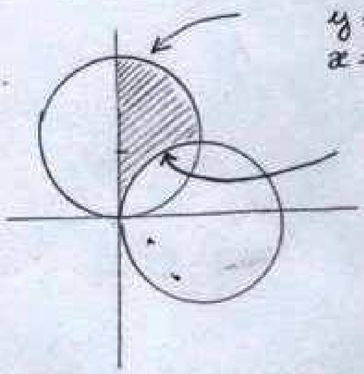
$$\int_0^1 \operatorname{sen} x^2 dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int_0^1 x^{4n+2} dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!\,(4n+3)}$$

Per l'osservazione al teorema di Leibniz sulle serie a segno alterno:

$|E_n| < \frac{1}{(2n+3)!\,(4n+7)} < 10^{-3}$ per $n \geq 1$

$\int_0^1 \operatorname{sen} x^2 dx \sim \frac{1}{3} - \frac{1}{42} \sim 0.309$ per difetto

2.



$y = 1 + \sqrt{1-x^2}$
$x = \sqrt{1-(y-1)^2}$

$y = \sqrt{1-(x-1)^2}$
$x = 1 - \sqrt{1-y^2}$

Area

$$\int_0^1 (1 - \sqrt{1-y^2})\, dy + \frac{\pi}{4} = 1$$

Come si può facilmente dedurre geometricamente

Volume : metodo gusci cilindrici

$$V = 2\pi \int_0^1 x(1+\sqrt{1-x^2})\, dx - 2\pi \int_0^1 x\sqrt{1-(x-1)^2}\, dx$$

$$= 2\pi \int_0^1 x\, dx + 2\pi \int_0^1 x\sqrt{1-x^2}\, dx - 2\pi \int_{-1}^0 \sqrt{1-x^2}\, dx - 2\pi \int_{-1}^0 x\sqrt{1-x^2}\, dx$$

$$= \frac{7}{3}\pi - \frac{\pi^2}{2}$$

Volume : metodo sezioni

$$V = \pi \int_0^1 (1 - \sqrt{1-y^2})^2 dy + \pi \int_1^2 (1-(y-1)^2)\, dy$$

$$= \pi \int_0^1 (2 - y^2 - 2\sqrt{1-y^2})\, dy + \pi \int_0^1 (1-y^2)\, dy$$

$$= \frac{7}{3}\pi - \frac{\pi^2}{2}$$

3.

$\frac{\lg t}{t(t-1)}$ no / si / si

$t \to 0$: $\sim \frac{-\lg t}{t} > \frac{1}{t}$ $t \to 1$: $\sim \frac{t-1}{t-1} \to 1$ $t \to \infty$: $\sim \frac{\lg t}{t^2} < \left(\frac{1}{t}\right)^{2-\alpha}$

scegliamo $\alpha > 0$: $2-\alpha > 1$ cioè $0 < \alpha < 1$

$F(x)$ C.E. $x > 0$

SGN $\underset{0}{\times}$ $-$ $\underset{1}{0}$ $+$

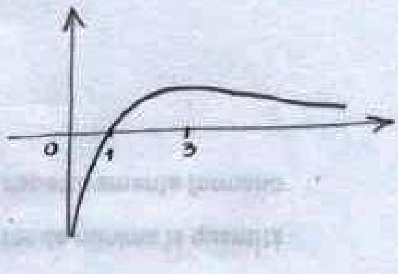
LIM per $x \to 0$ $F(x) \sim \int_x^{x^2} -\frac{\lg t}{t}\,dt = -\frac{1}{2}\left[\lg^2 t\right]_x^{x^2} = -\frac{3}{2}\lg^2 x \to -\infty$

oppure

$F(x) = \frac{(x^2-x)\lg \xi}{\xi(\xi-1)} \sim \frac{x^2 \lg \xi}{\xi^2}$

$\frac{x^2|\lg \xi|}{\xi^2} > \frac{x^2|\lg x|}{x^2} \to +\infty$

per $x \to +\infty$ $F(x) \to 0$



DRV

$F'(x) = \frac{(\lg x)(3-x)}{x(x^2-1)}$ $\underset{0}{+}$ $+$ $\underset{1}{1}$ $+$ $\underset{3}{0}$ $-$

4. Sl. omogenea : $y_0(x) = c_1 \cos x + c_2 \operatorname{sen} x$

Sl. particolare : det $W(x) = 1$

$W(x) = \begin{pmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{pmatrix}$

$c_1'(x) = \det\begin{pmatrix} 0 & \operatorname{sen} x \\ \frac{1}{\operatorname{sen} 2x} & \cos x \end{pmatrix} = -\frac{1}{2\cos x} \to c_1(x) = \frac{1}{2}\lg\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right|$

$c_2'(x) = \det\begin{pmatrix} \cos x & 0 \\ -\operatorname{sen} x & \frac{1}{\operatorname{sen} 2x} \end{pmatrix} = \frac{1}{2\operatorname{sen} x} \to c_2(x) = \frac{1}{2}\lg\left|\operatorname{tg}\frac{x}{2}\right|$

$\bar{y}_p(x) = \frac{1}{2}\cos x \lg\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + \frac{1}{2}\operatorname{sen} x \lg\left|\operatorname{tg}\frac{x}{2}\right|.$