Calcolo integrale – soluzioni degli esercizi proposti N. 1

N.B.: Tutte le soluzioni sono date a meno di una costante additiva arbitraria

1.

1. $e^{3x}\left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{27}\right)$
2. $x \operatorname{tg} x + \log|\cos x|$
3. $\frac{(x^2+1)\operatorname{arctg} x - x}{2}$
4. $-\frac{1}{2}e^{-x}(\operatorname{sen} x + \cos x)$
5. $x \log(x + \sqrt{1+x^2}) - \sqrt{1+x^2}$
6. $x \log^2 x - 2x(\log x - 1)$
7. $\frac{x^4 \log x}{4} - \frac{x^4}{16}$
8. $\left(\frac{x^3}{2} - \frac{3x}{4}\right)\operatorname{sen} 2x + \left(\frac{3x^2}{4} - \frac{3}{8}\right)\cos 2x$

2.

1. $\operatorname{sen} x - (\operatorname{sen}^3 x)/3$
2. $\operatorname{arctg}(x^2)/2$
3. $\log|\log x|$
4. $\log(1+e^x)$
5. $\operatorname{arctg}(\log x) - \frac{1}{2}\log(1+\log^2 x)$
6. $-\frac{1}{3}(2-x^2)^{3/2}$
7. $-\operatorname{arctg}\cos x$

3.

1. $\log\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}$
2. $-\frac{\sqrt{x^2+1}}{x}$
3. $2\sqrt{1+\operatorname{sen} x}$
4. $x + 4\sqrt{1+x} + 4\log\left|\sqrt{1+x}-1\right|$
5. $\frac{1}{4}\log\frac{1-\cos x}{1+\cos x} - \frac{1}{2}\frac{\cos x}{\operatorname{sen}^2 x}$
6. $\operatorname{tg}\frac{x}{2}$

7. $\operatorname{tg} x - x$

8. $2 \operatorname{arcsen} \frac{x}{2} + \frac{x \sqrt{4 - x^2}}{2}$

9. $\operatorname{arcsen} \frac{2x + 1}{3}$

10. $\log \left| \frac{1 - \sqrt{\frac{1 - x}{1 + x}}}{1 + \sqrt{\frac{1 - x}{1 + x}}} \right| + 2 \operatorname{arctg} \sqrt{\frac{1 - x}{1 + x}}$

4.

1. $x^2 - 4 \log (x^2 + 4) + \frac{1}{2} \operatorname{arctg} \frac{x}{2}$

2. $3 \log | x - 3 | - 2 \log | x - 2 |$

3. $\frac{1}{2} \log (x^2 - 2x + 4) + \sqrt{3} \operatorname{arctg} \frac{x - 1}{\sqrt{3}}$

4. $\log \frac{|x - 2|}{\sqrt{x^2 + 1}} - \operatorname{arctg} x - \frac{2}{x - 2}$

5. $2 \log (2x^2 - 5x + 7) + \frac{10}{\sqrt{31}} \operatorname{arctg} \frac{4x - 5}{\sqrt{31}}$

6. $\frac{1}{2} \log \left| \frac{x - 1}{x + 1} \right| - \frac{1}{2} \operatorname{arctg} x$

5.

1. $\frac{2}{1 - \operatorname{tg} (x / 2)}$

2. $\log | \operatorname{tg} x |$

3. $\frac{e^{\operatorname{tg} x}}{2}$

4. $- \log \left| \sqrt{x^2 - 4x + 5} - x + 2 \right|$

5. $\log \left| \frac{x - \sqrt{2x - x^2}}{x + \sqrt{2x - x^2}} \right|$

6. $\operatorname{tg} x - \frac{1}{2} \log (\operatorname{tg}^2 x + 2) + \sqrt{2} \operatorname{arctg} \frac{\operatorname{tg} x}{\sqrt{2}}$

7. $\frac{1}{5} \log \left| \frac{\operatorname{tg} \frac{x}{2} - \frac{1}{2}}{\operatorname{tg} \frac{x}{2} + 2} \right|$

8. $\frac{1}{2} \log \left| \frac{\operatorname{tg} x - 1}{\operatorname{tg} x + 1} \right|$

9. $\frac{x^2}{2} \operatorname{arcsen} x - \frac{1}{4} \operatorname{arcsen} x + \frac{1}{4} x \sqrt{1 - x^2}$