Analisi Matematica
Prova scritta del 4.2.10.

1. C.E. $\mathbb{R}$

SGN sempre positiva; si annulla per $x = 1$.

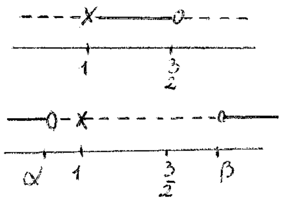
LIM per $x \to +\infty$, $f(x) \to 0$ ($y = 0$ asintoto orizz.)

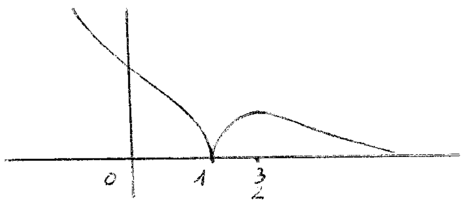
per $x \to -\infty$, $f(x) \to +\infty$. Poichè $f(x)/x \to -\infty$, non c'è asintoto.

DRV. $f'(x) = \operatorname{sgn}(x-1)\, e^{-x} \dfrac{3-2x}{2\sqrt{|x-1|}}$

$x = 1$ cuspide

$f''(x) = e^{-x} \dfrac{4x^2 - 12x + 7}{4|x-1|^{3/2}}$





2. $\displaystyle\int \frac{dy}{y} = \int \operatorname{arctg} \frac{x}{x-2}\, dx$

$\displaystyle\int \operatorname{arctg}\frac{x}{x-2}\, dx = x \operatorname{arctg}\frac{x}{x-2} + \int \frac{x}{x^2-2x+2}\, dx =$

$\displaystyle = x \operatorname{arctg}\frac{x}{x-2} + \frac{1}{2}\int \frac{2x-2}{x^2-2x+2}\, dx + \int \frac{dx}{(x-1)^2+1} =$

$\displaystyle = x \operatorname{arctg}\frac{x}{x-2} + \operatorname{arctg}(x-1) + \frac{1}{2}\lg(x^2-2x+2) + c = \varphi(x) + c$

$\lg|y| = \varphi(x) + c \longrightarrow |y| = K_+ e^{\varphi(x)}$ con $K_+ > 0 \longrightarrow$

$y = K e^{x \operatorname{arctg}\frac{x}{x-2}}\, e^{\operatorname{arctg}(x-1)} \sqrt{x^2-2x+2}$, con $K \in \mathbb{R}$.

La condizione iniziale è verificata per $K = e^{\pi/4}$.

3. Per $x \to 0^+$ $\dfrac{\operatorname{sen} x}{\lg(1+x)} \sim \dfrac{x}{x} \to 1$; per $x \to 0^-$ $e^{\frac{\lg\cos x}{x}} \sim e^{\frac{\lg(1-\frac{x^2}{2})}{x}} \sim e^{-\frac{x}{2}} \to 1$.

Si prolunga la funzione per continuità, ponendo $f(0) = 1$.

Per il rapporto incrementale si ha:

$\left(\dfrac{\operatorname{sen} x}{\lg(1+x)} - 1\right)/x = \dfrac{\operatorname{sen} x - \lg(1+x)}{x\lg(1+x)} \sim \dfrac{\frac{1}{2}x^2}{x^2} \to \frac{1}{2}$

$f'(0)$ non esiste
punto angoloso

$\left(e^{\frac{\lg\cos x}{x}} - 1\right)/x \sim \dfrac{\lg\cos x}{x^2} \sim \dfrac{\lg(1-\frac{1}{2}x^2)}{x^2} \sim \dfrac{-\frac{1}{2}x^2}{x^2} \to -\frac{1}{2}$

4. $\dfrac{a_{n+1}}{a_n} = \dfrac{3}{n+1}\left(1+\dfrac{1}{n}\right)^2 \dfrac{e^{\sqrt{n+1}}}{e^{\sqrt{n}}} \longrightarrow 0$ $\left(e^{\sqrt{n+1}} \sim e^{\sqrt{n}}\right)$

La serie converge.