Soluzioni [1]

1.

C.E. $x^2+x \geq 0 \iff x \leq -1 \vee x \geq 0$

SGN $f(x) \geq 0 \iff \sqrt{x^2+x} \geq x \iff x \leq -1 \vee \begin{cases} x \geq 0 \\ x^2+x \geq x^2 \end{cases} \iff$ sempre positiva nulla per $x=0$

LIM $x \to +\infty \quad f(x) = \frac{x}{\sqrt{x^2+x}+x} \sim \frac{x}{2x} \to \frac{1}{2}$ $\quad y=\frac{1}{2}$ asintoto orizz.

$x \to -\infty \quad f(x) \sim -2x \to +\infty, \; f(x)+2x = \sqrt{x^2+x}+x = \frac{x}{\sqrt{x^2+x}-x} \sim \frac{x}{-2x} \to -\frac{1}{2}$

$y = -2x - \frac{1}{2}$ as. obliquo

$f(0)=0, \; f(-1)=1$

DRV $f'(x) = \frac{2x+1}{2\sqrt{x^2+x}} - 1, \quad x \neq 0, x \neq -1$ (punti a tg. verticale)

$f'(x) \geq 0 \iff 2\sqrt{x^2+x} \leq 2x+1 \iff \begin{cases} x \geq 0 \\ 4x^2+4x \leq 4x^2+4x+1 \end{cases}$

$-1 \quad 0$

DRV² $f''(x) = -1/4(x^2+1)^{3/2} < 0$

La fz. $\sqrt{|x^2+x|} - x$ coincide con quella già studiata per $x \leq -1 \vee x \geq 0$.
Rimane da studiarla per $x \in (-1,0)$, dove può essere scritta $\sqrt{-x^2-x} - x$

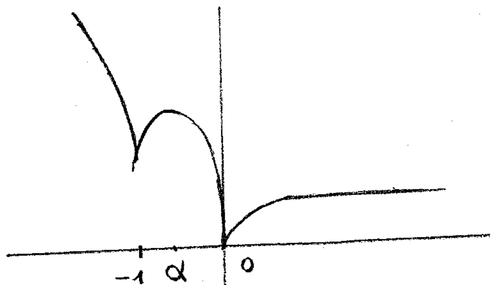
SGN Sempre positiva

DRV $f'(x) = \frac{-2x-1}{2\sqrt{-x^2-x}} - 1 \geq 0 \iff 2\sqrt{-x^2-x} \leq -2x-1 \iff \begin{cases} -1 < x \leq -\frac{1}{2} \\ 8x^2+8x+1 \geq 0 \end{cases}$

$-1 \quad \alpha \quad -\frac{1}{2} \quad 0$

$\alpha = -\frac{1}{2} - \frac{\sqrt{2}}{4}$

DRV² $f''(x) = -1/4(-x^2-x)^{3/2} < 0$



2. $\frac{\lg(1+x^2+\frac{x^4}{3}+o(x^4)) - 1 - x^2 - \frac{x^4}{2} + o(x^4) + 1}{1 - x^2 - \frac{1}{2}x^4 + o(x^4) - (1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4))^2} = \frac{x^2 + \frac{x^4}{3} - \frac{1}{2}x^4 - x^2 - \frac{1}{2}x^4 + o(x^4)}{1 - x^2 - \frac{x^4}{2} - 1 - \frac{x^4}{4} + x^2 - \frac{x^4}{12} + o(x^4)} =$

$\frac{-2/3 \, x^4 + o(x^4)}{-5/6 \, x^4 + o(x^4)} \longrightarrow \frac{4}{5}$

3. $\lg(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{6}{(1+c)^4 \, 4!} x^4 \qquad x = -1/5, \quad -1/5 < c < 0$

$\lg(8/10) \sim -\frac{1}{5} - \frac{1}{2}\left(\frac{1}{25}\right) - \frac{1}{3}\left(\frac{1}{125}\right) \sim -0{,}222$

$E = \frac{-6}{(1+c)^4 \, 4! \, 5^4} < 0$ appross. per eccesso $\qquad 1+c > 4/5, \quad \frac{1}{1+c} < \frac{5}{4}$

$|E| < \frac{6 \cdot 5^4}{4^4 \, 4! \, 5^4} = \frac{1}{4^5} < 10^{-3}$

1.

C.E. $\quad x^2 - x \geq 0 \iff x \leq 0 \vee x \geq 1$

SGN $\quad f(x) \geq 0 \iff \sqrt{x^2-x} \geq -x \iff x \geq 1 \vee \begin{cases} x \leq 0 \\ x^2 - x \geq x^2 \end{cases} \iff$ sempre positiva nulla per $x = 0$

LIM $\quad$ per $x \to +\infty \quad f(x) \sim 2x \to +\infty$, $f(x) - 2x = \dfrac{-x}{\sqrt{x^2-x} + x} \sim \dfrac{-x}{2x} \to -\dfrac{1}{2} \qquad y = 2x - \frac{1}{2}$ asintoto

per $x \to -\infty \quad f(x) = \dfrac{-x}{\sqrt{x^2-x} - x} \sim \dfrac{-x}{-2x} \to \dfrac{1}{2} \qquad y = \frac{1}{2}$ asintoto orizz.

$f(0) = 0 \qquad f(1) = 1$

DRV $\quad f'(x) = \dfrac{2x-1}{2\sqrt{x^2-x}} + 1 \qquad x \neq 0,\ x \neq 1$ (punti a tg. verticale)

$f'(x) \geq 0 \iff 2\sqrt{x^2-x} \geq 1 - 2x \iff x \geq 1 \vee \begin{cases} x \leq 0 \\ 4x^2 - 4x < 4x^2 - 4x + 1 \end{cases} \iff x \geq 1$

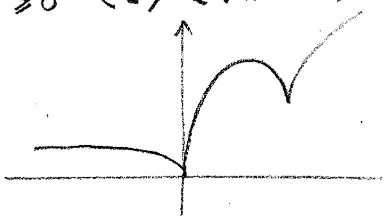
$$\text{---}\times \quad \times\text{———} \qquad 0 \quad 1$$

DRV^2 $\quad f''(x) = -1/4(x^2-x)^{3/2} < 0$

Per $x \in (0,1)$ studiamo la fz. $\sqrt{x - x^2} + x$.

SGN $\quad$ sempre positiva

DRV $\quad f'(x) = \dfrac{1-2x}{2\sqrt{x-x^2}} + 1 \geq 0 \iff 2\sqrt{x-x^2} \geq 2x - 1 \iff 0 < x \leq \frac{1}{2} \vee \begin{cases} \frac{1}{2} < x < 1 \\ 8x^2 - 8x + 1 \leq 0 \end{cases}$



$$\times\text{———}\circ\text{- - -}\times \qquad 0 \quad \tfrac{1}{2} \quad \alpha \quad 1$$

$\alpha = \frac{1}{2} + \frac{\sqrt{2}}{4}$

2.

$$\frac{3\left(1 + \left(x - \frac{x^3}{6} + o(x^4)\right)^2\right)^{1/3} - 5 + 2\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)\right)}{1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{x^2}{2} - 1 + o(x^4)}$$

$$\frac{3\left(1 + x^2 - \frac{x^4}{3} + o(x^4)\right)^{1/3} - 5 + 2 - x^2 + \frac{1}{12}x^4 + o(x^4)}{-\frac{x^4}{8} + o(x^4)} =$$

$$\frac{3\left(1 + \frac{x^2}{3} - \frac{x^4}{9} - \frac{x^4}{9}\right) - 3 - x^2 + \frac{x^4}{12} + o(x^4)}{-\frac{x^4}{8} + o(x^4)} = \frac{-\frac{7}{12}x^4 + o(x^4)}{-\frac{1}{8}x^4 + o(x^4)} \to \frac{14}{3}$$

3. $\quad \sqrt[3]{1+x} = 1 + \dfrac{x}{3} - \dfrac{x^2}{9} + \dfrac{5x^3}{81} - \dfrac{80}{81}\dfrac{1}{(1+c)^{11/3}}\dfrac{x^4}{24} \qquad x = \frac{1}{8} \qquad 0 < c < \frac{1}{8}$

$\sqrt[3]{9} \sim 2\left(1 + \frac{1}{3}\left(\frac{1}{8}\right) - \frac{1}{9}\left(\frac{1}{64}\right) + \frac{5}{81}\left(\frac{1}{512}\right)\right) \sim 2{,}080$

$E = -\dfrac{80}{81}\dfrac{1}{(1+c)^{11/3}}\dfrac{1}{8^4\,24} < 0$ appross. per eccesso $\qquad 1 + c > 1 \qquad \dfrac{1}{1+c} < 1$

$|E| < \dfrac{80}{81 \cdot 8^4 \cdot 24} \sim 10^{-5}$

1. C.E. $x^2+2x \geq 0 \iff x \leq -2 \vee x \geq 0$

SGN $f(x) \geq 0 \iff \sqrt{x^2+2x} \geq x \iff x \leq -2 \vee \begin{cases} x \geq 0 \\ x^2+2x \geq x^2 \end{cases} \iff$ sempre positiva, nulla per $x=0$

LIM per $x \to +\infty$ $f(x) = \dfrac{2x}{\sqrt{x^2+2x}+x} \sim \dfrac{2x}{2x} \to 1$ $\quad y=1$ asintoto orizz.

per $x \to -\infty$ $f(x) \sim -2x \to +\infty$, $f(x)+2x = \dfrac{2x}{\sqrt{x^2+2x}-x} \sim \dfrac{2x}{-2x} \to -1$ $\quad y=-2x-1$ asintoto

DRV $f'(x) = \dfrac{x+1}{\sqrt{x^2+2x}} - 1$, $x \neq 0, x \neq -2$ (punti a tangente verticale)

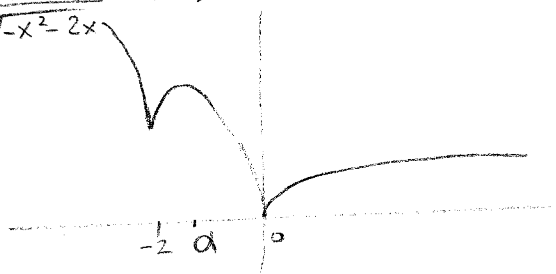
$f'(x) \geq 0 \iff \sqrt{x^2+2x} \leq x+1 \iff \begin{cases} x \geq 0 \\ x^2+2x \leq x^2+2x+1 \end{cases}$ $\quad -2 \quad 0$

DRV^2 $f''(x) = -1/(x^2+2x)^{3/2} < 0$

Per $x \in (-2,0)$ studiamo la fz. $\sqrt{-x^2-2x} - x$.

SGN sempre positiva

DRV $f'(x) = -\dfrac{x+1}{\sqrt{-x^2-2x}} - 1 \geq 0 \iff \sqrt{-x^2-2x} \leq -x-1 \iff \begin{cases} -2 < x \leq -1 \\ x^2+4x+1 \geq 0 \end{cases}$ $\quad -2 \quad \alpha \quad -1 \quad 0$ $\quad \alpha = -1-\frac{1}{\sqrt{2}}$



2. $\dfrac{-x^2-\frac{x^4}{2}-2+2\left(1+\frac{1}{2}x^2-\frac{1}{8}x^4\right)+o(x^4)}{1+x^2+\frac{x^4}{2}-3+2\left(1-\frac{1}{2}x^2+\frac{1}{24}x^4\right)+o(x^4)} = \dfrac{-\frac{3}{4}x^4+o(x^4)}{\frac{7}{12}x^4+o(x^4)} \to -\dfrac{9}{7}$

3. $\lg(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{6}{(1+c)^4\, 4!} x^4$ $\quad x = -\dfrac{1}{10}$, $\quad -\dfrac{1}{10} < c < 0$

$\lg(9/10) \sim -\dfrac{1}{10} - \dfrac{1}{2}\left(\dfrac{1}{100}\right) - \dfrac{1}{3}\left(\dfrac{1}{1000}\right) \sim -0{,}1053$ $\quad 1+c > \dfrac{9}{10}$, $\dfrac{1}{1+c} < \dfrac{10}{9}$

$E = -\dfrac{6}{(1+c)^4\, 4!\, 10^4} < 0$ appross. per eccesso

$|E| < \dfrac{6 \cdot 10^4}{9^4\, 4!\, 10^4} = \dfrac{1}{4 \cdot 9^4} < 4\ 10^{-5}$

Soluzioni [4]

1.
C.E. $\quad x^2-2x \geq 0 \iff x \leq 0 \vee x \geq 2$

SGN $\quad f(x)\geq 0 \iff \sqrt{x^2-2x} \geq -x \iff x\geq 2 \vee \begin{cases} x\leq 0 \\ x^2-2x\geq x^2\end{cases} \iff$ sempre positiva, nulla per $x=0$

LIM $\quad$ per $x\to+\infty$ $\quad f(x)\sim 2x\to+\infty$

$$f(x)-2x = \sqrt{x^2-2x}-x = \frac{-2x}{\sqrt{x^2-2x}+x} \sim \frac{-2x}{-2x}\to 1 \qquad y=2x+1 \text{ asintoto}$$

per $x\to-\infty$ $\quad f(x)=\dfrac{-2x}{\sqrt{x^2-2x}-x} \sim \dfrac{-2x}{-2x}\to 1 \qquad y=1$ asintoto orizz.

DRV
$$f'(x)=\frac{x-1}{\sqrt{x^2-2x}}+1 \qquad x\neq 0,\ x\neq 2 \text{ (punti a tg. verticale)}$$

$$f'(x)\geq 0 \iff \sqrt{x^2-2x}\geq 1-x \iff x\geq 2 \vee \begin{cases} x\leq 0\\ x^2-2x\geq x^2-2x+1\end{cases}$$

$0 \quad 2$

DRV^2 $\quad f''(x) = -1/(x^2-2x)^{3/2}<0$

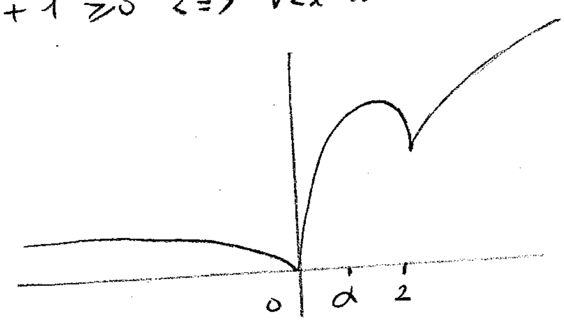
Per $x\in(0,2)$ studiamo la fz. $\sqrt{2x-x^2}+x$.

SGN $\quad$ sempre positiva

DRV $\quad f'(x)=\dfrac{1-x}{\sqrt{2x-x^2}}+1\geq 0 \iff \sqrt{2x-x^2}\geq x-1 \iff x\leq 1 \vee \begin{cases}1<x<2\\ 2x^2-4x+1\leq 0\end{cases}$

$0 \quad 1 \quad \alpha \quad 2$

$\alpha = 1+\frac{1}{\sqrt{2}}$



2.
$$\frac{x^2-\frac{x^4}{2}+o(x^4)-2+2\left(1-\frac{x^2}{2}-\frac{x^4}{8}+o(x^4)\right)}{1+x^2+\frac{x^4}{2}-2+1-2x^2+\frac{2}{3}x^4+\left(x+\frac{x^3}{3}+o(x^4)\right)^2} =$$

$$\frac{-\frac{3}{4}x^4+o(x^4)}{-x^2+\frac{7}{6}x^4+x^2+\frac{2}{3}x^4+o(x^4)} = \frac{-\frac{3}{4}x^4+o(x^4)}{\frac{11}{6}x^4+o(x^4)} \to -\frac{9}{22}$$

3.
$$\sqrt[3]{1+x} = 1+\frac{x}{3}-\frac{x^2}{9}+\frac{5}{81}x^3-\frac{80}{81}\frac{1}{(1+c)^{11/3}}\frac{x^4}{24} \qquad x=\frac{1}{27},\ 0<c<\frac{1}{27}$$

$$\sqrt[3]{28} = 3\left(1+\frac{1}{3}\left(\frac{1}{27}\right)-\frac{1}{9}\left(\frac{1}{27}\right)^2+\frac{5}{81}\left(\frac{1}{27}\right)^3\right)\sim 3,036$$

$1+c>0, \quad \frac{1}{1+c}<1$

$E = -\dfrac{80}{81}\dfrac{1}{(1+c)^{11/3}}\dfrac{1}{27^4\, 24}<0$ $\quad$ appross. per eccesso

$$|E| < \frac{80}{81}\frac{1}{27^4\cdot 24} < 8\cdot 10^{-8}$$