Verificare le seguenti uguaglianze dopo aver stabilito per quali valori di x sono valide

$$\log_{10}\frac{\sqrt[3]{x^4-x}}{3-\sqrt{x+1}} = \log_{10}x + \frac{1}{3}\log_{10}(x-1) - \log_{10}(3-\sqrt{x+1})$$

$1<x<8$ $\left(]1;8[\right)$

$$\log_5\sqrt{\frac{5^x\cdot 25^x}{5(|x|-\sqrt{2(x^2-1)})}} = \frac{3}{2}x - \frac{1}{2} - \frac{1}{2}\log_5|x| - \frac{1}{2}\log_5\left(1-\sqrt{2-\frac{2}{x^2}}\right)$$

$-\sqrt{2}<x\le -1 \vee 1\le x<\sqrt{2}$

$\left(]-\sqrt{2};-1]\cup[1;\sqrt{2}[\right)$

Equazioni e disequazioni esponenziali.

- $a^{f(x)} = a^{g(x)} \iff f(x)=g(x) \quad \forall a>0 \quad a\neq 1$
- $a^{f(x)} \le a^{g(x)} \iff \begin{cases} f(x)\le g(x), & a>1 \\ f(x)\ge g(x), & 0<a<1 \end{cases}$
- $a^{f(x)} \ge a^{g(x)} \iff \begin{cases} f(x)\ge g(x), & a>1 \\ f(x)\le g(x), & 0<a<1 \end{cases}$

$\left(\frac{2}{3}\right)^x = \frac{27}{8}$; $x=-3$ $\qquad \left(\sqrt{0,1}\right)^x = 100 \quad x=-4$

$4^{\frac{3+x}{x-1}} = 2^{5x} \quad x=2 \vee x=-\frac{3}{5}$

$3^{4+x}\le 9 \quad]-\infty;-2]$; $\quad 9^{4-x}\ge\frac{1}{3} \quad]-\infty;\frac{9}{2}]$

$\left(\frac{1}{2}\right)^{x-2}<4 \quad]0;+\infty[$; $\quad \frac{1}{2^{x^2}}<\frac{1}{4} \quad]-\infty;-\sqrt{2}[\cup]\sqrt{2};+\infty[$

$7^{\sqrt{x^2-1}}\le 49 \quad [-\sqrt{5};-1]\cup[1;\sqrt{5}]$; $\quad \left(\frac{1}{5}\right)^{|x+4|}>25 \quad \emptyset$

$\left(\frac{1}{3}\right)^{|2x-1|}<\frac{1}{27}$; $]-\infty;-1[\cup]2;+\infty[$

$2^{2\sqrt{|x+2|}} \geq 16 \quad]-\infty; -6] \cup [2; +\infty[$

$2^{|x|-1} + 1 < \frac{4}{3} \quad]-\frac{1}{2}; \frac{1}{2}[$

$\frac{5^{|x+2|} - 5}{e^x - \sqrt{e}} \leq 0 \quad]-\infty, -3] \cup [-1; \frac{1}{2}[$

$a^x = b \iff \begin{cases} x = \log_a b = \frac{\log b}{\log a} & \text{se } b > 0 \\ \emptyset & \text{se } b \leq 0 \end{cases}$

$\boxed{a > 0, a \neq 1}$

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per $a > 1$ e $b > 0$

$\left[\begin{array}{ll} a^x < b \iff & x < \log_a b \\ a^x > b \iff & x > \log_a b \end{array}\right.$

per $0 < a < 1$ e $b > 0$

$\left[\begin{array}{ll} a^x < b \iff & x > \log_a b \\ a^x > b \iff & x < \log_a b \end{array}\right.$

per $a > 0, a \neq 1$ e $b \leq 0$

$\left[\begin{array}{ll} a^x > b, & \forall x \in \mathbb{R} \\ a^x < b, & \emptyset \end{array}\right.$

$9^x = 5 \quad x = \frac{\ln 5}{2 \ln 3} \qquad \left(\frac{1}{2}\right)^x = \sqrt{3} \quad x = -\frac{1}{2} \frac{\ln 3}{\ln 2}$

$|2^{x+1} - 3| = 2 \quad x = \frac{\ln 5 - \ln 2}{\ln 2} \lor x = -1$

$2 \cdot 3^x = 4^{x+1} \; ; \; x = -\frac{\ln 2}{2 \ln 2 - \ln 3}$

$3^{1-x} - 3^{1+x} = 8 \; ; \; x = -1$

$3 \cdot 2^{x \log_2 9} + 2^{x \log_2 3} = 2 \; ; \; x = -1 + \frac{\ln 2}{\ln 3}$

$3^x - 2 > 0$ $\quad \left]\frac{\ln 2}{\ln 3}; +\infty\right[$; $\quad \left(\frac{1}{4}\right)^x - 5 > 0$ $\quad \left]-\infty; -\frac{\ln 5}{\ln 4}\right[$

$2^x + 1 \leq 0$, $\emptyset$; $\quad \left(\frac{1}{3}\right)^x + \left(\frac{1}{9}\right) > 0 \quad \forall x \in \mathbb{R}$

$3^x - 7^x \leq 0$, $[0; +\infty[$; $\quad 6^x - 3^{2x+1} < 0$, $\left]-\frac{\ln 3}{\ln 3 - \ln 2}; +\infty\right[$

$5^{1+\sqrt{x}} - 5^{2-\sqrt{x}} < 20 \quad [0; 1[$

$$\frac{3 \cdot 3^x + 3^{2-x} - 4}{3^x} < \frac{8}{3} \quad]1; 2[$$

$$|3^{2x} - 2| < 3 \quad \left]-\infty; \frac{1}{2}\frac{\ln 5}{\ln 3}\right[$$

$$\frac{e^x + 1 - 2e^{-x}}{e^{2x} - 3} \geq 0 \quad]-\infty; 0] \cup \left]\frac{1}{2}\ln 3; +\infty\right[$$

Equazioni e Disequazioni logaritmiche.

$a > 0 \quad a \neq 1 \qquad \log_a x = b \iff x = a^b, \ \forall b \in \mathbb{R}$

$$a > 1, \ b \in \mathbb{R} \quad \begin{cases} \log_a x < b \iff 0 < x < a^b \\ \log_a x > b \iff x > a^b \end{cases}$$

$$0 < a < 1, \ b \in \mathbb{R} \quad \begin{cases} \log_a x < b \iff x > a^b \\ \log_a x > b \iff 0 < x < a^b \end{cases}$$

$\log_4 x = 2 \quad x = 16$; $\quad \log_{\frac{1}{3}} x = -3 \quad x = 27$

$\ln x = \frac{1}{2} \quad x = \sqrt{e}$; $\quad \log_{\frac{4}{3}} x = 0 \quad x = 1$

$\log_{\frac{1}{2}} x = -\frac{2}{3} \quad x = \sqrt[3]{4}$; $\quad \log_2 x = -\frac{3}{2} \quad x = \frac{\sqrt{2}}{4}$

$\ln^2 x = 1 \quad x = \frac{1}{e} \vee x = e$; $\quad 2\ln^2 x - \ln x^3 - 2 = 0$

$x = e^2 \vee x = \frac{\sqrt{e}}{e}$

$\log_{\frac{1}{3}} \log_{\frac{1}{3}}(2x+1) = 0 \qquad x = -\frac{1}{3}$

$\log_{\sqrt{2}} x + 2 < 0 \quad]0; \frac{1}{2}[\quad ; \quad \log_{\frac{1}{4}} x < \frac{3}{2} \quad]\frac{1}{8}; +\infty[$

$\log_{\frac{4}{3}} x - 1 > 0 \quad]\frac{4}{3}; +\infty[\quad ; \quad \log_{\frac{3}{4}} x > -2 \quad]0; \frac{16}{9}[$

$|\log_2 x + 2| < 1 \quad]\frac{1}{8}; \frac{1}{2}[\quad ; \quad 2\log_e^2 x - \log_e \frac{1}{x} > 0 \quad]0; \frac{\sqrt{e}}{e}[\cup$
$\cup]1; +\infty[$

$2(\log_{\frac{1}{2}} x)^2 + \log_{\frac{1}{2}} x \geq 1 \qquad]0; \frac{\sqrt{2}}{2}] \cup [\sqrt{2}; +\infty[$

$\sqrt{1 + \log_{\sqrt{2}} x} < 2 \quad ; \quad [\frac{\sqrt{2}}{2}; 2\sqrt{2}[\quad ; \quad |3\ln^2 x + 5\ln x| < 2$
$]\frac{1}{e^2}; \frac{\sqrt[3]{e}}{e}[\cup]\frac{1}{e}; \sqrt[3]{e}[$

$\log_2^2 |x| + 2\log_2 |x| - 3 \geq 0 \quad]-\infty; -2] \cup [-\frac{1}{8}; 0[\cup]0; \frac{1}{8}] \cup [2; +\infty[$

$\sqrt{1 - 2\ln x} > \sqrt{3}\ln x \qquad]0; \sqrt[3]{e}[$

$\log_{\frac{1}{2}}(2^x - 1) \geq -2 \quad]0; \frac{\ln 5}{\ln 2}]$

$\log(|x| - 2) < 1 \quad]-2-e; -2[\cup]2; 2+e[$

$\log_2(3 - \log_{\frac{1}{2}} x) + \log_{\frac{1}{3}} 9 \leq 0 \qquad]\frac{1}{8}; 2]$

$\frac{\log_e(\sqrt[3]{|x|} - 1)}{\log_e^2 x - \log_e x} \geq 0 \qquad]1; e[\cup [8; +\infty[$

$\sqrt{\ln x} \leq 1 - \ln(x^2) \qquad [1; \sqrt[4]{e}]$

$\frac{2\log_2(2 - |x|) - 1}{\sqrt{3 - e^x} - 1} \geq 0 \qquad [-(2-\sqrt{2}); (2-\sqrt{2})] \cup]\ln 2; \ln 3]$

$$\log_a f(x) = \log_a g(x) \iff \begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ f(x) = g(x) \end{cases} \qquad \forall\, a > 0,\ a \neq 1$$

$$\log_a f(x) < \log_a g(x) \iff \begin{cases} f(x) > 0 \\ f(x) < g(x) \end{cases} \qquad \text{se } a > 1$$

$$\log_a f(x) < \log_a g(x) \iff \begin{cases} g(x) > 0 \\ f(x) > g(x) \end{cases} \qquad \text{se } 0 < a < 1$$

$$\log_a f(x) > \log_a g(x) \iff \begin{cases} g(x) > 0 \\ f(x) > g(x) \end{cases} \qquad \text{se } a > 1$$

$$\log_a f(x) > \log_a g(x) \iff \begin{cases} f(x) > 0 \\ f(x) < g(x) \end{cases} \qquad \text{se } 0 < a < 1$$

$\log_2 (3x - 5) = 1 - \log_2 (x - 2)$ $\qquad x = \frac{8}{3}$

$\log_{\frac{1}{2}} \sqrt{3x-2} - \log_{\frac{1}{2}} \sqrt{7-x} = \log_{\sqrt{3}} \frac{1}{\sqrt{3}}$ $\qquad x = \frac{30}{7}$

$\log_{10} \sqrt{3-x} < \frac{1}{2} \log_{10} (3x-4)$ $\qquad]\frac{7}{4}; 3[$

$2 \log_{\frac{1}{2}} \sqrt{2x} < \log_{\frac{1}{2}} (x^2 - 4)$ $\qquad]2; 1+\sqrt{5}[$

$2 \log_2 |x| - 2 \log_2 (3 - x^2) + \log_2 (x^2 + 3) \leq 0$ $\qquad [-1; 0[\cup]0; 1]$

$\log_2 (x^2 - 3x) - \log_2 (1 - x) < 1$ $\qquad]-1; 0[$

$\log_{\sqrt{2}} (3x - 5) + \log_{\sqrt{2}} (x - 2) - 2 < 0$ $\qquad]2; \frac{8}{3}[$

$\log_{\frac{1}{2}} (3x - 1) > \log_{\frac{1}{4}} (2x + 1)$ $\qquad]\frac{1}{3}; \frac{8}{9}[$

$\log_{\sqrt{2}} \frac{1+|x|}{1-|x|} + \log_{\frac{1}{3}} 2 > 0$ $\qquad]-1; -\frac{1}{3}[\cup]\frac{1}{3}; 1[$

$\ln (2e^x + 1) > \ln (4 - e^{2x})$ $\qquad]0; \ln 2[$