RELATIVE SET THEORY

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Relative set theory postulates that the universe of mathematical entities is stratified into **levels** by means of a primitive binary predicate $x \in \mathbf{V}(y)$ (x *appears at the level of* y). In [1], elementary analysis is developed in the framework of the relative set theory FRIST [2], an extension of RIST of Péraire [3]. This development will be described by Richard O'Donovan in his talk. I will discuss GRIST [4], a further extension of FRIST, and its metamathematical properties.The (internal) methods of traditional nonstandard analysis can be formalized in set theory with just two levels, *standard* and *internal*. Among such theories, BST of [5] is singled out by a number of pleasing properties, among them by being, in a precise sense, the theory of the "universal limit ultrapower of the universe," and "complete over ZFC." Every countable model $\mathcal{M}$ of ZFC has a unique (up to isomorphism) extension to a countable model $\mathcal{N}$ of BST in which $\mathcal{M}$ is the class of standard sets. I will present analogous results for GRIST. In these results, the place of ultrafilters is taken by *stratified ultrafilters*, roughly, ultrafilters over a set of ultrafilters over ... a set of principal ultrafilters, iterated into transfinite.Some of the extra strength of GRIST is useful in applications, for example, the "relative" treatment of the generalized Riemann integral given in [1].

References

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