ALGEBRAS OF GENERALIZED FUNCTIONS AND NONSTANDARD ANALYSIS

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[Joint work with Todor Todorov.]

The Colombeau algebra of generalized functions provides a framework to treat nonlinear partial differential equations which involve singular data, where classical distribution theory (in the sense of L. Schwartz) cannot be applied, due to a lack of well-defined nonlinear operations on the space of distributions. We show how ultrafilters can be used to construct an algebra of generalized functions endowed with a canonical embedding of the space of Schwartz distributions, similar to Colombeau's algebra, but with improved algebraic properties. E.g., the set of scalars of our algebra is a totally ordered, algebraically closed field unlike its counterpart in Colombeau theory, which is a ring with zero divisors. As a consequence, a Hahn-Banach extension theorem holds which does not hold in Colombeau theory. The constructed algebra has a natural interpretation within nonstandard analysis.

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