Max e Min assoluti

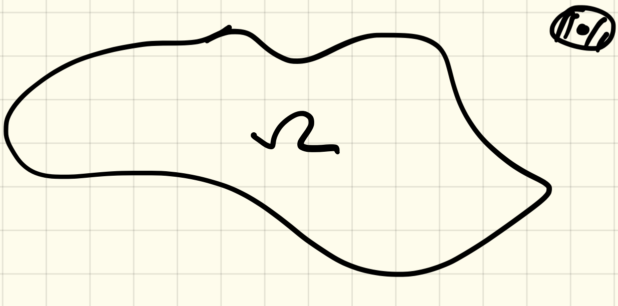
$\Omega \subseteq \mathbb{R}^n$

$f : \Omega \longrightarrow \mathbb{R}$

$\exists \ \max\limits_{\Omega} f$ e $\min\limits_{\Omega} f$.

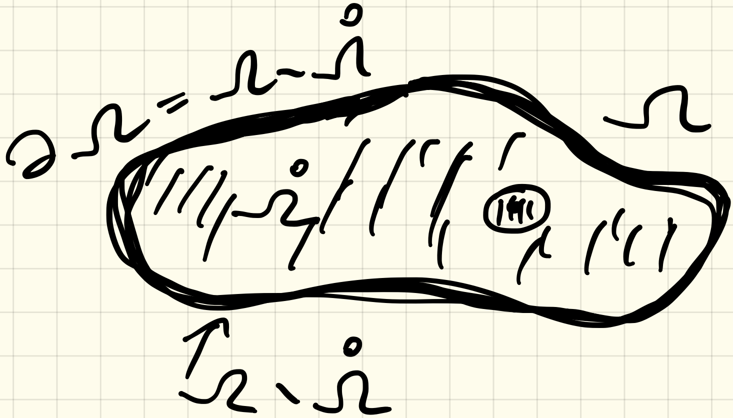
RISPOSTA (Weierstrass)

O.K. se Ω è chiuso e limitato ed f è continua.



COME TROVARE MAX e MIN ?

$$\Omega \to \mathring{\Omega}$$
$$\Omega \to \Omega \setminus \mathring{\Omega}$$



1° PASSO $\quad \nabla f(x) = \vec{0} \in \mathbb{R}^n$

$\{P_1, \dots, P_m\}$ i punti di $\mathring{\Omega}$ dove si annulla il ∇f.

2° PASSO Studiare

$$\max_{\Omega \setminus \mathring{\Omega}} f \quad \text{e} \quad \min_{\Omega \setminus \mathring{\Omega}} f$$

3 PASSO Si fa confronto tra i valori

$$\left\{ \underbrace{f(P_1), \ldots, f(P_n)}, \underbrace{\max_{\partial \Omega} f}, \underbrace{\min_{\partial \Omega} f} \right\}$$

$(n+2)$ valori

Il più grande tra questi numeri è il max assoluto

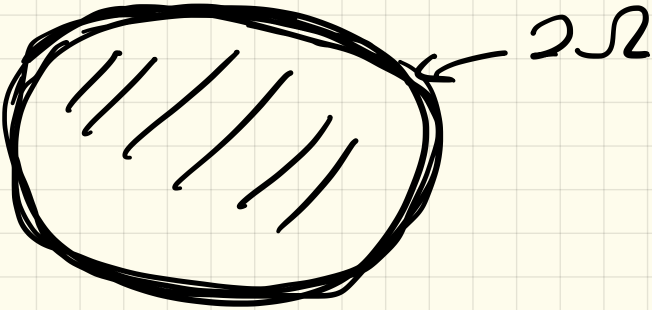
Il più piccolo sarà il minimo assoluto.

Come calcolare $\max_{\Omega\setminus\mathring{\Omega}} f$ e $\min_{\Omega\setminus\mathring{\Omega}} f$

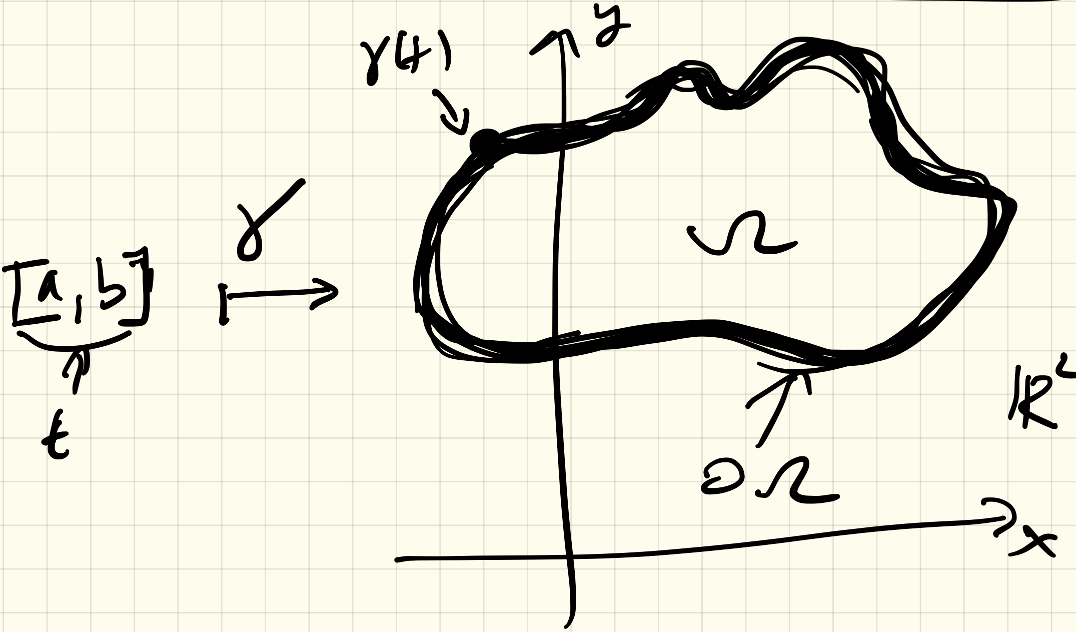
<u>oss.</u> Se $n=1$

$$[a,b] \longrightarrow \partial([a,b]) = \{a,b\}$$

<u>oss.</u> Se $n>1$ le cose sono più complicate



METODO DI PARAMETRIZZAZIONE



Supponiamo di avere

$\gamma : [a,b] \longrightarrow \mathbb{R}^2$

t.c.

$\gamma(t) \in \partial\Omega \quad \forall t$

$\gamma([a,b]) = \partial\Omega$

Se ho γ (che chiamo parametrizzazione di $\partial\Omega$)
allora

$$\operatorname*{Max}_{\partial\Omega} f = \operatorname*{Max}_{t\in[1,5]} \boxed{f\circ(\gamma(t))}$$

$$\operatorname*{Min}_{\partial\Omega} f = \operatorname*{Min}_{t\in[1,5]} \boxed{f\circ(\gamma(t))}$$

Esempio

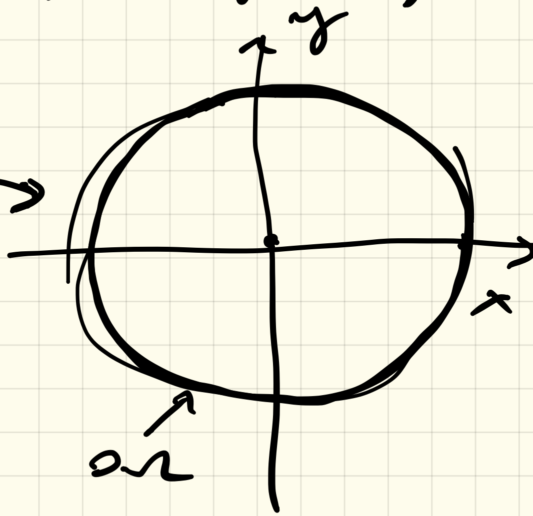
$$\Omega = \{ x^2 + y^2 \le 1 \}$$

chiuso e limitato

$$\mathring{\Omega} = \{ x^2 + y^2 < 1 \}$$

$$\partial\Omega = \{ (x,y) \mid x^2 + y^2 = 1 \}$$

$$[a,b] \xrightarrow{\gamma}$$



$[a,b] = [0, 2\pi]$

$$[0, 2\pi] \xrightarrow{\gamma} (\cos t, \sin t)$$

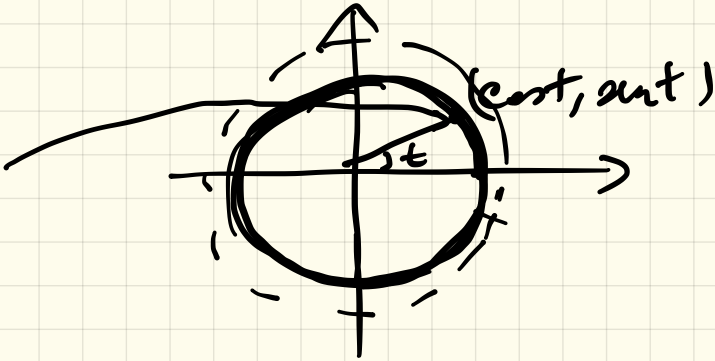
$\uparrow$

t

t

$\in$

$[0, 2\pi]$



$$\underbrace{f(\cos t, \sin t)}_{\uparrow} \quad \text{cont } t \in \underline{[0, 2\pi]}$$

Esercizio

Calcolare

$$\max_K f \quad e \quad \min_K f$$

dove $f(x,y) = \underline{x^2 - y^2}$

$K = \{(x,y) \mid x^2+y^2 \leq 1\}$.

$\overset{\circ}{K} = \{(x,y) / x^2+y^2 < 1\}$

$\partial K = \{(x,y) / x^2+y^2 = 1\}$

$\nabla f = (2x, -2y)$

$$\begin{cases} 2x = 0 \\ -2y = 0 \end{cases} \Longrightarrow (x,y) = \boxed{(0,0)} \in \overset{\circ}{K}$$

Studiare $\underline{\max_{\partial K} f}$ e $\underline{\min_{\partial K} f}$

$\max_{\partial K} f$ e $\min_{\partial K} f$.

$\boxed{\text{NOT.}}$ In genere Ω indica un aperto.

In genere si preferisce indicare con K un chiuso e limitato.

$\partial K = \{ x^2+y^2=1 \}$

$[0,2\pi] \ni t \longrightarrow (\cos t, \operatorname{sen} t)$

è una parametrizzazione di ∂K

$f(\cos t, \operatorname{sen} t) = \cos^2 t - \operatorname{sen}^2 t$

$$\underset{t\in[0,2\pi]}{\mathrm{Max}}\,\underbrace{(\cos^2 t - \operatorname{sen}^2 t)}_{} \longrightarrow \underbrace{\underset{\partial K}{\mathrm{Max}}\, f}_{}$$

$$\underset{t\in[0,2\pi]}{\mathrm{Min}}\,\underbrace{(\cos^2 t - \operatorname{sen}^2 t)}_{} \longrightarrow \underbrace{\underset{\partial K}{\mathrm{Min}}\, f}_{}$$

$$\cos^2 t - \operatorname{sen}^2 t = \boxed{\underbrace{\cos(2t)}_{}}$$

$$t = 0 \longrightarrow \cos(2t) = \boxed{1}$$

$$t = \frac{\pi}{2} \longrightarrow \cos(2t) = \boxed{-1}$$

$$\left\{ f(0,0),\ 1,\ -1 \right\} = \underbrace{\left\{ 0, 1, -1 \right\}}_{}$$

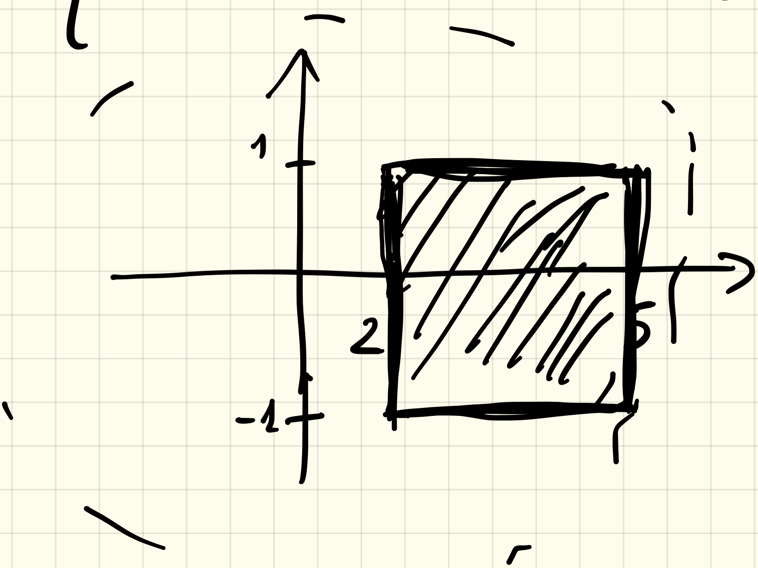
$$\Longrightarrow \boxed{\underset{K}{\mathrm{Max}}\, f = 1, \quad \underset{K}{\mathrm{Min}}\, f = -1}$$

Esempio

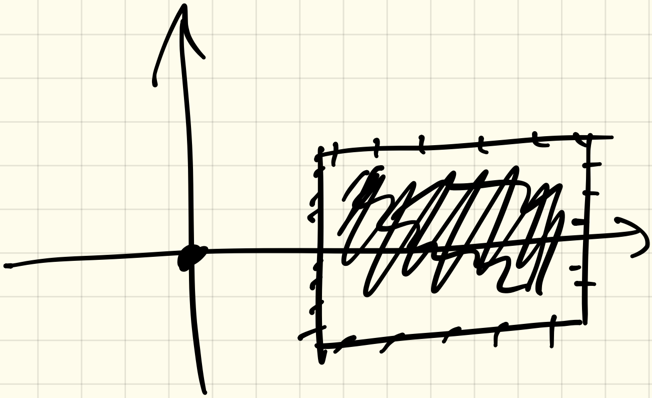
$$\max_K f \quad e \quad \min_K f$$

$$f(x,y) = 2x^2 + 3y^2$$

$$K = \{(x,y) \mid 2 \le x \le 5, -1 \le y \le 1\}$$



$\mathring{K}$



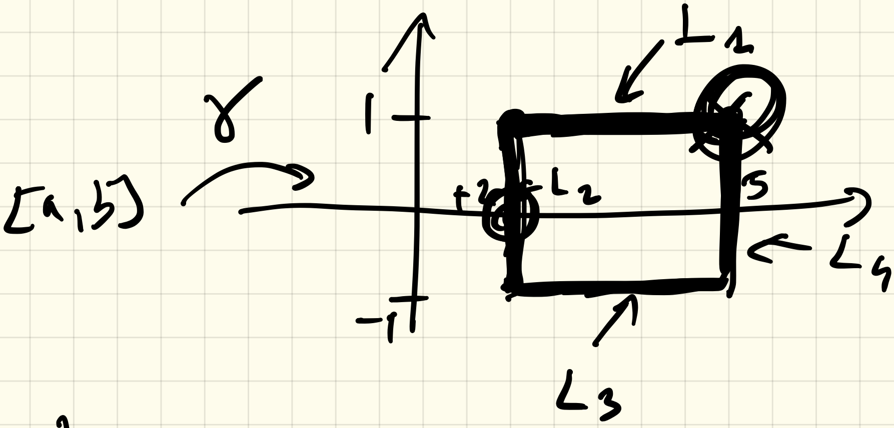
$$\nabla f = (0,0)$$

$$\nabla f = (4x, 6y)$$

$$\begin{cases} 4x = 0 \\ 6y = 0 \end{cases} \Rightarrow (x,y) = (0,0) \cancel{A}$$

$\mathring{K}$

Studiamo ∂K



$$\text{Max}\left\{\underbrace{\text{Max}_{L_1} f}, \underbrace{\text{Max}_{L_2} f}, \underbrace{\text{Max}_{L_3} f}, \underbrace{\text{Max}_{L_4} f}\right\}$$

$$\parallel$$

$$\text{Max}_{\partial K} f$$

$$\text{Min}\left\{\text{Min}_{L_1} f, \text{Min}_{L_2} f, \text{Min}_{L_3} f, \text{Min}_{L_4} f\right\}$$

$$\parallel$$

$$\text{Min}_{\partial K} f$$

$$\max_{L_1} f \quad e \quad \min_{L_1} f \qquad f(x,y) = 2x^2 + 3y^2$$

$$\gamma_1 : \underset{[2,5]}{t} \longrightarrow (t, 1) \in L_1$$

$$f(\gamma_1(t)) = 2t^2 + 3$$

$$\max_{[2,5]} 2t^2 + 3 = \boxed{53}$$

$$\min_{[2,5]} 2t^2 + 3 = \boxed{11}$$

Calcoliamo

$$\max_{t\in[2,5]} 2t^2+3 = \boxed{53}$$

$$(2t^2+3)' = 4t = 0 \iff \boxed{t=0}$$

$$\min_{t\in[2,5]} 2t^2+3 = \boxed{11}$$

Lato L_2 $\quad f(x,y)=2x^2+3y^2$

$$\gamma_2 : [-1,1] \ni t \longrightarrow (2,t)$$

$$f \circ \gamma_2(t) = 8 + 3t^2$$

$$\max_{t \in [-1,1]} 8+3t^2 = \boxed{11}$$

$$\min_{t \in [-1,1]} 8+3t^2 = \boxed{8}$$

$$(8+3t^2)' = 6t = 0 \iff \boxed{t=0}$$

$$\{8, 11, 11\}$$

L_3

$$\gamma_3 : [2,5] \ni t \longrightarrow (t, -1)$$

$$f \circ (\gamma_3(t)) = 2t^2 + 3$$

$$\max_{[2,5]} 2t^2 + 3 = \boxed{53}$$

$$\min_{[2,5]} 2t^2 + 3 = \boxed{11}$$

4.4

$$[-1,1] \ni t \xrightarrow{\gamma_4} (5, t)$$

$$f \circ (\gamma_4(t)) = 50 + 3t^2$$

$$\max_{[-1,1]} 50 + 3t^2 = \boxed{53}$$

$$\min_{[-1,1]} 50 + 3t^2 = \boxed{50}$$

$$(50 + 3t^2)' = 6t = 0 \iff \boxed{t = 0}$$

$$\{50, 53, 53\}$$

$$\max_{\partial K} f = \boxed{53}$$

$$\min_{\partial K} f = \boxed{8}$$

Siccome non abbiamo punti interni in cui si annulla il gradiente

$$\{8, 53\}$$

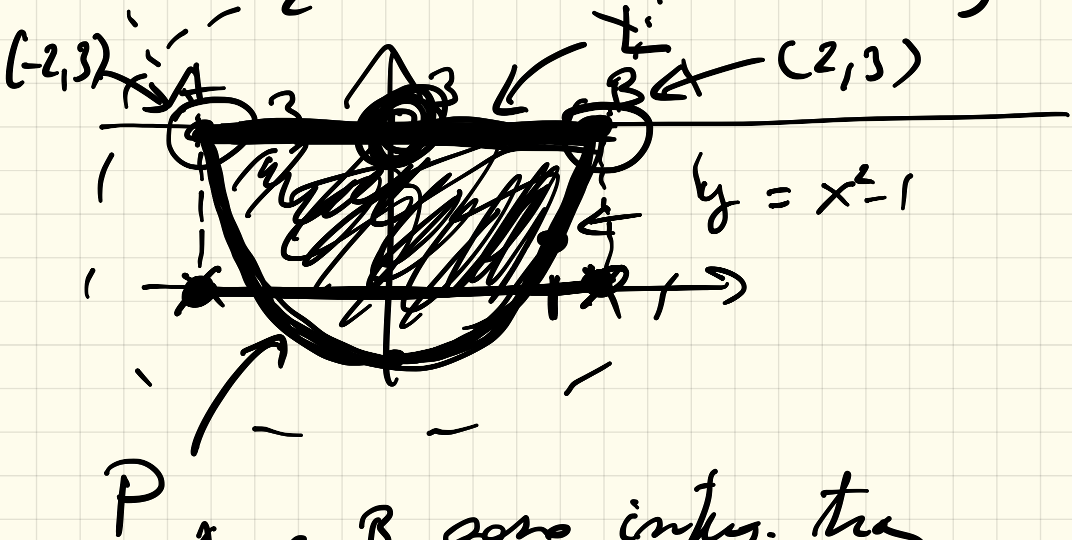
$$\Rightarrow \boxed{\begin{array}{l} \max_{K} f = 53 \\ \min_{K} f = 8 \end{array}}$$

Esercizio

$\max_K f$ e $\min_K f$

$f(x,y) = 3x^2 - y + 3$

$K = \{(x,y) \in \mathbb{R}^2 / x^2 - 1 \leq y \leq 3\}$



A e B sono inters. tra parabola e retta $y = 3$

$3 = x^2 - 1 \iff x^2 = 4 \iff x = \pm 2$

K°

$$\nabla f = (0,0)$$

$$(6x, -1)$$

$$\Rightarrow \{\nabla f = (0,0)\} = \emptyset$$

$$\partial K = P \cup L$$

Studio $\max_L f$ e $\min_L f$.

$f = 3x^2 - y + 3$

$\gamma_L : [-2,2] \ni t \longrightarrow (t, 3)$

$f \circ \gamma_L(t) = 3t^2$

$\max_{[-2,2]} 3t^2 = \boxed{12}$

$\min_{[-2,2]} 3t^2 = \boxed{0}$

$(3t^2)' = 6t = 0 \iff \boxed{t = 0}$

$\{0, 12, 12\}$

$\underset{p}{\mathrm{Max}}\, f$ e $\underset{p}{\mathrm{Min}}\, f$

$$f(x,y) = 3x^2 - y + 3$$

$$\gamma_p : [-2,2] \ni t \longrightarrow (t\,,\, t^2-1)$$

$$f \circ (\gamma_p(t)) = 3t^2 - t^2 + 1 + 3$$

$$= \boxed{2t^2 + 4}$$

$$\underset{[-2,2]}{\mathrm{Max}}\; 2t^2+4 = \boxed{12}$$

$$\underset{[-2,2]}{\mathrm{Min}}\; 2t^2+4 = \boxed{4}$$

$$\{4,\ 12,\ 12\}$$

$$(2t^2+4)' = 4t = 0 \iff \boxed{t=0}$$

$$\max_{\partial K} f = \boxed{12}$$

$$\min_{\partial K} f = \boxed{0}$$

$$\max_{K} f = \boxed{12}$$

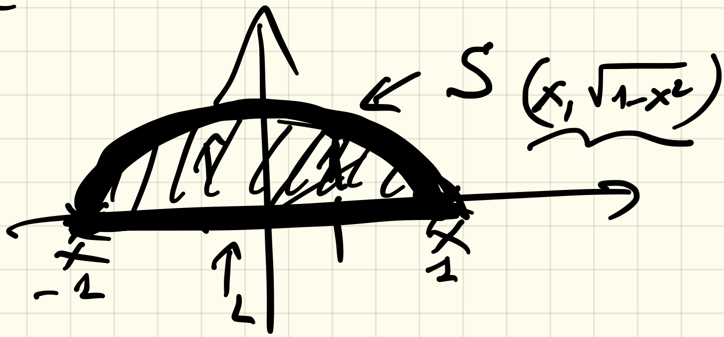
$$\min_{K} f = \boxed{0}$$

ESERCIZIO

$\max_K f$ e $\min_K f$

$f(x,y) = \underline{x^2 - y^2}$

$K = \{(x,y) / x^2+y^2 \leq 1,\ y \geq 0\}$



$\underline{\gamma_S} : \underbrace{[0,\pi]} \ni t \longrightarrow \underbrace{(\cos t, \operatorname{sen} t)}$

$\underline{\text{Oss.}}$ Altra param. di S

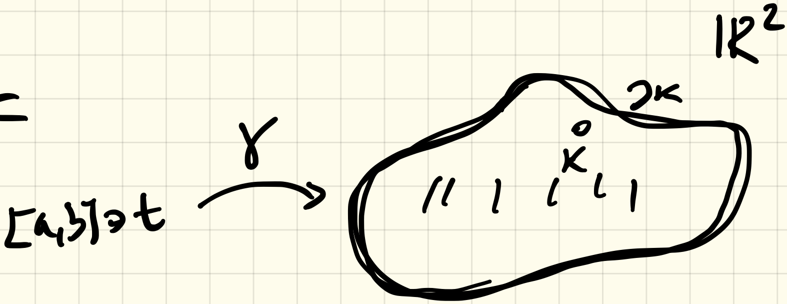
$\underline{\gamma_S} : \underbrace{[-1,1]} \ni t \longrightarrow \underbrace{(t, \sqrt{1-t^2})}$

Per studiare $\max_{\partial K} f$ e $\min_{\partial K} f$

può essere utile la

<u>parametrizzazione</u>.

<u>Idea</u>

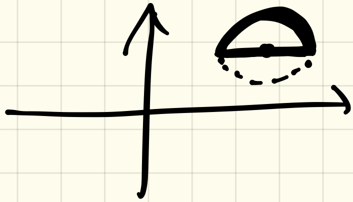


$$\max_{t\in[a,b]} f\circ\gamma(t) \quad e \quad \min_{t\in[a,b]} f\circ\gamma(t)$$

<u>Oss.</u> A volte ∂K si può spezzare in più pezzi ad ognuno dei quali applichiamo la param

Come parametrizzare
circonf di raggio R e centro (x_0, y_0)

$t \in [0, 2\pi] \longrightarrow (x_0 + R\cos t,\ y_0 + R \,\mathrm{sen}\, t)$

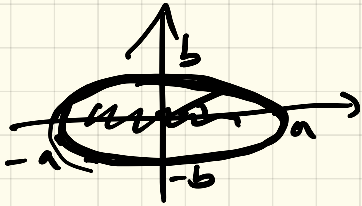


$t \in [0, \pi] \longrightarrow$

Parametrizzazione dell'ellisse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$[0, 2\pi] \ni t \xrightarrow{\gamma}$$



$$\gamma(t) = (a\cos t, b\sin t) \in \underline{\text{Ellisse}}$$

Se fossi interessato

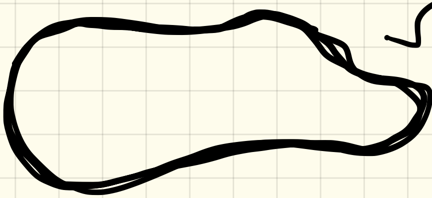
$$t \in \left[0, \frac{\pi}{2}\right], \quad \gamma(t) = (a\cos t, b\sin t)$$

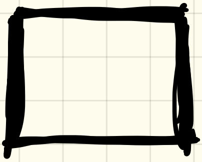
MOLTIPLICATORI DI LAGRANGE

Utile per studiare Max f e Min f

OK OK

Debolezza metodo parametrizzazione

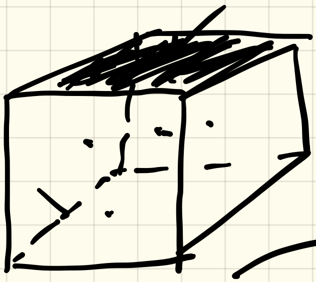
$t \xrightarrow{\gamma}$  $\Omega \subseteq \mathbb{R}^2$

$[a,b] \ni t \xrightarrow{\gamma}$ 

$$\Omega \subseteq \mathbb{R}^3$$

$$\max_K f(x,y,z) \quad e \quad \min_K f(x,y,z)$$

K è un cubo 3-dimensionale

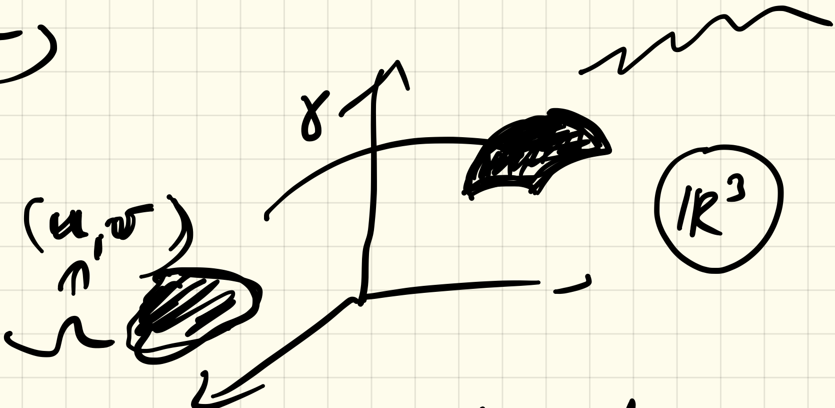


$$(\partial_x f, \partial_y f, \partial_z f)$$

$$\mathring{K} \longrightarrow \nabla f = (0,0,0)$$

∂K è fatto da 6 facce
ognuna delle quali ha dim 2!

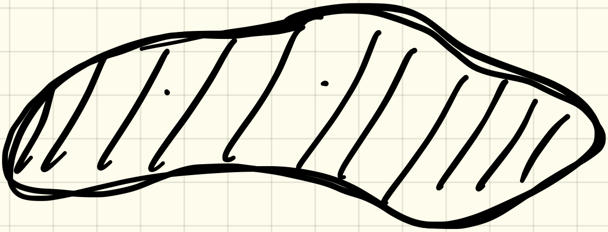
Quindi non posso parametrizzare
la faccia con la sola variabile
t! mi servono due variabili
(u,v)



Quindi $f \circ \gamma(u,v)$ diventa

una funzione di 2 variabili
da massimizzare o minimizzare
in

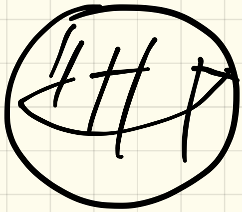
$$\Omega \subseteq \mathbb{R}^2$$



$\subseteq \mathbb{R}^2$

MOLTIPLICATORI DI LAGRANGE

$f(x_1,\dots,x_n)$, $K \subseteq \mathbb{R}^n$



Come procedo (usando Lagrange?)

<u>1° PASSO</u> $\nabla f = (0,\dots,0) \in \mathbb{R}^n$

$\{P_1,\dots,P_k\}$ punti di $\mathring{K}$ dove si annulla ∇f.

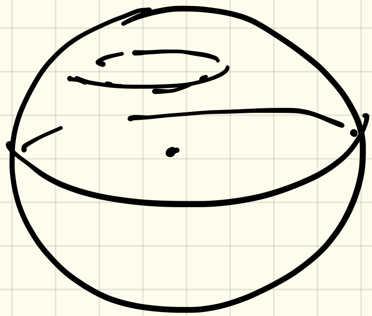
2° PASSO $\quad \max\limits_{\partial K} f(x_1, \dots, x_n)$ e

$$\min_{\partial K} f(x_1, \dots, x_n).$$

Lagrange (moltiplicatori) permettono di studiare questo problema a patto che

$$\partial K = \{(x_1, \dots, x_n) \in \mathbb{R}^n / G(x_1, \dots, x_n) = 0\}$$

Es. $K = \{(x,y,z) \in \mathbb{R}^2 \mid x^2+y^2+z^2 \le 1\}$



$$\mathring{K} = \{(x,y,z) \mid x^2+y^2+z^2 < 1\}$$

$$\partial K = \{(x,y,z) \mid \underline{x^2+y^2+z^2 = 1}\}$$

$$\boxed{G(x,y,z) = \underline{x^2+y^2+z^2-1}}$$

$$\Longrightarrow \partial K = \{(x,y,z) / \underbrace{G(x,y,z)} = 0\}$$

Siamo quindi nel caso in cui vogliamo studiare:

$$\max_{\{G(x_1,\dots,x_n)=0\}} \boxed{f(x_1,\dots,x_n)} \quad e$$

$$\min_{\boxed{\{G(x_1,\dots,x_n)=0\}}} f(x_1,\dots,x_n)$$

$G \rightsquigarrow \partial K$

$f \rightsquigarrow$ funzione da ottimizzare

Metodo Moltiplicatori

1° PASSO

$$(1)\begin{cases} \nabla G = 0 \\ G(x) = 0 \end{cases}$$

$\nabla G = 0 \leftarrow$ n equazioni e $x \in \partial K \subset \mathbb{R}^n$

$G(x) = 0 \leftarrow$ 1 eq.

(n+1) eq. in n incognite

Quindi in generale mi aspetto poche o nessuna sol.

Se ci sono sol. $\boxed{\{P_1, \ldots, P_k\}}$

2° PASSO

moltiplicatori di Lagrange

$$(2)\begin{cases}\nabla f = \lambda \nabla G \\ G(x) = 0\end{cases}$$

n eq. in $(n+1)$ incognite $(x_1, \dots, x_n, \lambda)$

$(n+1)$ eq. in $(n+1)$ incognite $(x_1, \dots, x_n, \lambda)$

Risolvo questo sistema;

$\{Q_1, Q_2, \dots, Q_k\}$ sol. di questo sistema

$$Q_i = (x_1^i, \dots, x_n^i, \lambda^i) \longrightarrow$$

$$\tilde{Q}_i = (x_1^i, \dots, x_n^i)$$

$$\{\tilde{Q}_1, \tilde{Q}_2, \dots, \tilde{Q}_h\}$$

Faccio alla fine confronto

$$\max_{\partial K} f = \max\{f(P_1), \dots, f(P_k), f(\tilde{Q}_1), \dots, f(\tilde{Q}_h)\}$$

$$\min_{\partial K} f = \min\{ \text{-- -- ——} \quad \text{-- -- ——} \}$$

Esempio

$$\boxed{(0,0)}$$

$$\max_{\{(x,y)\,|\,x^2+y^2\le 1\}=K} (x^2-y^2) \quad , \quad \min_{\{(x,y)\,|\,x^2+y^2\le 1\}} (x^2-y^2)$$

1° PASSO $\quad \nabla f = (2x, -2y)$

$$\begin{cases} 2x = 0 \\ -2y = 0 \end{cases} \iff (x,y) = (0,0) \in \mathring{K}$$

$$\mathring{K} = \{(x,y) \,|\, x^2+y^2 < 1\}$$

2° PASSO

$$\underset{\partial K}{\mathrm{Max}}\,(x^2-y^2)\ , \quad \underset{\partial K}{\mathrm{Min}}\,(x^2-y^2)$$

$$\partial K = \boxed{\{(x,y) \mid x^2+y^2=1\}}$$

parametrizzazione

Lagrange

$$\partial K = \{(x,y) \mid x^2+y^2-1=0\}$$

quindi $G(x,y) = \boxed{x^2+y^2-1}$

Scriviamo i due sistemi di Lagrange:

$$f(x,y) = x^2 - y^2$$

$$G(x,y) = x^2 + y^2 - 1$$

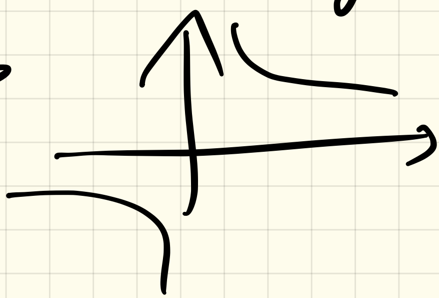
$$(1) \begin{cases} 2x = 0 \\ 2y = 0 \\ x^2 + y^2 - 1 = 0 \end{cases} \Longrightarrow \begin{array}{l} x = 0 \\ y = 0 \\ 0^2 + 0^2 - 1 \neq 0 \end{array}$$

$\boxed{\text{(NO SOL.)}}$

Se $G(x,y)$ fosse $x^2 - y^2 - 1$

$$x^2 - y^2 - 1 = 0$$

$$x^2 - y^2 = 1$$



(2) $\begin{cases} \nabla f(x,y) = \lambda \nabla G(x,y) \\ G(x,y) = 0 \end{cases}$

$f = x^2 - y^2$

$\nabla G = (2x, 2y)$

$G = \boxed{x^2 + y^2 - 1}$

$\nabla f = (2x, -2y)$

$$\begin{cases} 2x = \lambda 2x \\ -2y = \lambda 2y \\ x^2 + y^2 - 1 = 0 \end{cases} \longrightarrow \underbrace{3 \text{ eq.}}_{\boxed{(x, y, \lambda)}}$$

$$\boxed{G(x_1, \dots, x_n) = 0, \ G(x) = 0, \ G = 0}$$

$$\begin{cases} 2x = \lambda 2x & \rightsquigarrow \; 2x\cdot(1-\lambda) = 0 \\ -2y = \lambda 2y & \rightsquigarrow \; 2y(\lambda+1) = 0 \\ x^2+y^2 = 1 & \rightsquigarrow \; x^2+y^2 = 1 \end{cases}$$

Dalla prima eq.

$$x = 0 \xrightarrow{3^a \text{ eq.}} y^2 = 1 \longrightarrow y = \boxed{\pm 1}$$

$$(0, 1, -1) \qquad (0, -1, -1)$$

$$\boxed{\lambda = 1} \xrightarrow{2^a \text{ eq.}} 2y\cdot 2 = 0 \longrightarrow y = 0$$

$$\xrightarrow{3^a \text{ eq.}} x^2 = 1 \longrightarrow x = \pm 1$$

$$(1, 0, 1) \qquad (-1, 0, 1)$$

$\tilde{Q}_i$

$(0,1),\ (0,-1),\ (1,0),\ (-1,0)$

secondo sistema di Lagrange

$(0,0)$ ← interno

min min max max

$$\{ f(0,1), f(0,-1), f(1,0), f(-1,0), f(0,0) \} =$$

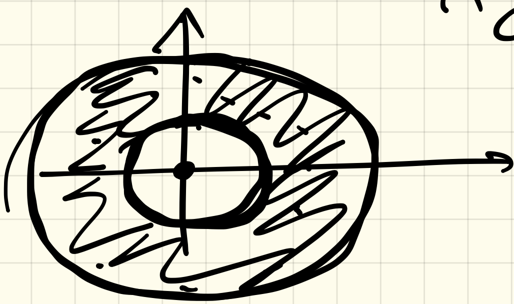
$$= \{ -1, -1, 1, 1, 0 \}$$

Esercizio

$$\max_{K} (x^2 - y^3) \qquad \min_{K} (x^2 - y^3)$$

$$K = \{ (x,y) \mid 1 \le x^2 + y^2 \le 4 \}$$

Chiuso e limitato



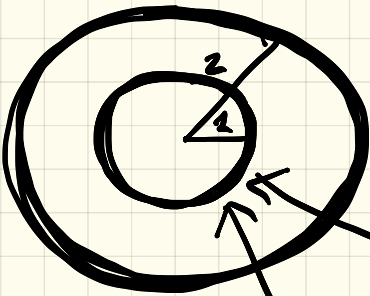
1° PASSO $\overset{\circ}{K}$

$$\nabla f = (0,0) \iff (2x, -3y^2) = (0,0)$$

$$\Longrightarrow \boxed{(x,y) = (0,0)} \notin \overset{\circ}{K}$$

2° PASSO Studio Max x^2-y^3 e Min x^2-y^3

ok ok



C_1 param. → $(2\cos t, 2\sin t)$

$\{x^2+y^2-4=0\}$

$(\cos t, \sin t)$

$\{x^2+y^2-1=0\}$ Lagr.

C_2

Il bordo è fatto da due pezzi

$\boxed{\begin{array}{l} \text{Max } (x^2-y^3) \\ C_1 \\ \text{Max } (x^2-y^3) \\ C_2 \end{array}}$ e $\boxed{\begin{array}{l} \text{Min } (x^2-y^3) \\ C_1 \\ \text{Min } (x^2-y^3) \\ C_2 \end{array}}$

Param. m $\boxed{C_1}$

$$\gamma(t) = (2\cos t, 2\sin t)$$

$t \in [0, 2\pi]$

$$f \circ \gamma(t) = 4\cos^2 t - 8\sin^3 t$$

$$\max_{t \in [0, 2\pi]} (4\cos^2 t - 8\sin^3 t) \leftarrow$$

$$\min_{t \in [0, 2\pi]} (4\cos^2 t - 8\sin^3 t) \leftarrow$$

Studiem prind

$$(4\cos^2 t - 8\sin^3 t)' = 0 \longrightarrow$$

... ---- -

FACCIAMO LAGRANGE
SU C_2

$$f(x,y) = \underbrace{x^2 - y^3}$$

$$G(x,y) = \underbrace{x^2 + y^2 - 1}$$

$$(1) \begin{cases} 2x = 0 \\ 2y = 0 \\ x^2 + y^2 - 1 = 0 \end{cases} \rightsquigarrow \nexists$$

$$\boxed{(2)\ \begin{cases} 2x = \lambda \cdot 2x \\ -3y^2 = \lambda \cdot 2y \\ x^2+y^2-1=0 \end{cases}}$$

$$\downarrow$$

$$\begin{cases} 2x(1-\lambda)=0 \\ y\cdot(2\lambda+3y)=0 \\ x^2+y^2-1=0 \end{cases}$$

$x=0 \xrightarrow{3^a\,eq.} y^2=1$

$y=\pm 1$

$\boxed{(0,\pm 1)}$

$\lambda = 1$

$\xrightarrow{2^a\,eq.}$ $y\cdot(2+3y)=0 \longrightarrow$ $\boxed{y=0}$, $\boxed{y=-\frac{2}{3}}$

$y=0 \xrightarrow{3^a\,eq.} \boxed{x=\pm 1}$ $\boxed{(\pm 1, 0)}$

$y=-\frac{2}{3} \xrightarrow{3^a\,eq.} x^2+\frac{4}{9}-1=0 \quad x^2 = 1-\frac{4}{9} = \frac{5}{9}$

$$\boxed{\underbrace{\left(\pm\frac{\sqrt5}{3},\ -\frac{2}{3}\right)},\ (0,\pm1),\ (\pm1,0)}$$

$$f(0,\pm1) = \boxed{\pm1} \quad -1 \text{ min on } \boxed{C_2}$$

$$f(\pm1,0) = \boxed{1} \longleftarrow \text{ max on } \boxed{C_2}$$

$$f\underbrace{\left(\pm\frac{\sqrt5}{3},-\frac{2}{3}\right)} = \boxed{\frac{5}{9}+\frac{8}{27}} \qquad C_2^-$$

$$\| $$

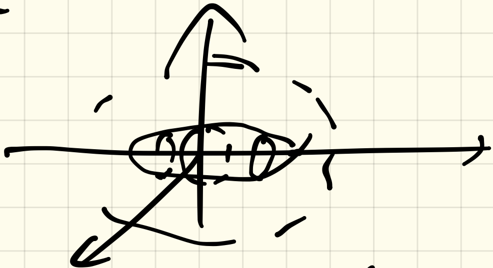
$$\frac{15+8}{27} = \frac{23}{27} < 1$$

Esercizio

$$f(x,y,z)$$

$$\max_K \left(x^2-2y+3xz\right) \quad e \quad \min_K \left(x^2-2y+3xz\right)$$

$$K=\left\{(x,y,z) \mid x^2+2y^2+3z^2 \leq 7\right\}$$



$$\overset{\circ}{K}=\left\{x^2+2y^2+3z^2<7\right\}$$

$$\partial K=\left\{x^2+2y^2+3z^2-7=0\right\}$$

$$\boxed{G(x,y,z)=x^2+2y^2+3z^2-7}$$

1° PASSO $(\mathring{K})$

$$2x+3z=0$$
$$-2=0 \leftarrow \text{impos.}$$

2° PASSO (∂K)

$$(1)\ \begin{cases} 2x = 0 \\ 4y = 0 \\ 6z = 0 \end{cases} \Rightarrow \boxed{(0,0,0)}$$

$$x^2+2y^2+3z^2=7 \leftarrow \text{Impos.}$$

(1) non ha sol.

(2) $\begin{cases} 2x+3z = \lambda \cdot 2x \\ -2 = \lambda \cdot 4y \leftarrow \boxed{\lambda \neq 0} \\ 3x = \lambda \cdot 6z \\ x^2+2y^2+3z^2 = 7 \end{cases}$

Eq. 2 $\Longrightarrow \boxed{\lambda \neq 0}$

Posso quindi ricavare z dalla 3ª eq. ${}^{2}\cancel{6}\lambda z = \cancel{3}x \Rightarrow 2\lambda z = x$

$\Rightarrow \boxed{z = \frac{x}{2\lambda}}$ o.k. perché $\boxed{\lambda \neq 0}$

1ª eq. $\Rightarrow \boxed{2x + \frac{3x}{2\lambda} = 2\lambda x}$

$$2x + \frac{3x}{2\lambda} - 2\lambda x = 0$$

$$\boxed{\left(0, \pm\frac{\sqrt{7}}{\sqrt{2}}, 0\right)}$$

$$\Downarrow$$

$$x \cdot \left(2 + \frac{3}{2\lambda} - 2\lambda\right) = 0$$

$$\boxed{x = 0}$$

$$\boxed{2 + \frac{3}{2\lambda} - 2\lambda = 0}$$

$\downarrow$ $\boxed{\text{eq. } 3}$

$$z = 0$$

$\downarrow$ eq. 4

$$\boxed{2y^2 = 7} \rightarrow y = \pm\frac{\sqrt{7}}{\sqrt{2}}$$

$$2 + \frac{3}{2\lambda} - 2\lambda = 0$$

$$\Updownarrow$$

$$4\lambda + 3 - 4\lambda^2 = 0 \xRightarrow{\text{eq. 2° grado}} \boxed{\lambda = -\frac{1}{2}, \frac{3}{2}}$$

$$\boxed{\begin{matrix} \left(\frac{\sqrt{5}}{2}, 1, -\frac{\sqrt{5}}{2}\right) \\ \left(-\frac{\sqrt{5}}{2}, 1, \frac{\sqrt{5}}{2}\right) \end{matrix}}$$

$$\boxed{\lambda = -\frac{1}{2}} \longrightarrow -2 = 4\lambda y = -2y$$

$$\longrightarrow \boxed{y = 1}$$

$$2x + 3z = -x \longrightarrow \boxed{3x + 3z = 0}$$

$$3x = -3z \longrightarrow \boxed{x = -z}$$

4ª eq.

$$x^2 + 2 + 3x^2 = 7$$

$$4x^2 = 5 \qquad x = \pm\frac{\sqrt{5}}{2}$$

$$\boxed{\lambda = \frac{3}{2}} \longrightarrow$$

$\boxed{\lambda = \frac{3}{2}} \longrightarrow -2 = 4\lambda y \Rightarrow -2 = 6y$

$\Rightarrow \boxed{y = -\frac{1}{3}}$

$3x = 6\lambda z \Rightarrow \boxed{3x = 9z} \Rightarrow \boxed{x = 3z}$

4ª eq.

$\left(\frac{3}{\sqrt{2}}, -\frac{1}{3}, \frac{1}{\sqrt{2}}\right)$

$\left(-\frac{3}{\sqrt{2}}, -\frac{1}{3}, -\frac{1}{\sqrt{2}}\right)$

$$9z^2 + 2 \cdot \frac{1}{9} + 3z^2 = 7$$

$$12z^2 = 7 - \frac{2}{9} = \frac{54}{9} = 6$$

$\boxed{z^2 = \frac{1}{2}}$