Esercizi sul calcolo di Max e Min assoluti

Calcolare ad occhio"(cioe' senza fare conti) i seguenti $\max_A f$ e $\min_A f$ dove:

(1)

$$A = [1,2] \times [2,3], f(x,y) = x + \frac{1}{y}$$

[R: $2 + 1/2, 1 + 1/3$];

(2)

$$A = \{(x,y,z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 \leq 100\}, f(x,y,z) = \sin(x^2 + y^2 + z^2)$$

[R: $1, -1$];

(3)

$$A = \{(x,y) \in \mathbb{R}^2 | |x+y| \leq 5, |x| \leq 2, f(x,y) = |\sin(x+y)||\cos x|$$

[R: $1, 0$];

(4)

$$A = \{(x,y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1, |x| \leq 1/2\}, f(x,y) = x^2 + y^2 - 1$$

[R:$0, -1$]

Calcolare usando qualsiasi metodo (ossia: parametrizzazione, moltiplicatori di Lagrange oppure entrambi) i seguenti $\max_A f$ e $\min_A f$ dove:

(1)

$$A = \{(x,y) \in \mathbb{R}^2 | x + y \leq 3, x \geq 1/2, y \geq 1/2\}$$
$$f(x,y) = x + y + \frac{1}{x} + \frac{1}{y}$$

[R: $\frac{27}{5}, 4$]

(2)

$$A = \{(x,y) \in \mathbb{R}^2 | 1 \leq x^2 + y^2 \leq 4\}$$
$$f(x,y) = x^2 - y^3$$

[R: $8, -8$]

(3)

$$A = \{(x,y) \in \mathbb{R}^2 | |x| + |y| \leq 1$$
$$f(x,y) = (x^2 + y^2)e^{x-y}$$

[R: $e, 0$]

(4)

$$A = \{(x,y) \in \mathbb{R}^2 | x \in [0, \frac{\pi}{2}], y \in [0, \frac{\pi}{2}]\}$$
$$f(x,y) = \sin x + \sin y + \sin(x+y)$$

[R: $\frac{3\sqrt{3}}{2}, 0$]

(5)

$$f(x,y) = x^2 + 5y^2 - \frac{xy}{2}$$
$$A = \{(x,y) \in \mathbb{R}^2 | x^2 + 4y^2 \leq 4\}$$

[R: $9/2 + \sqrt{2}/2, 0$]

(6)

$$f(x,y) = xye^{-xy}$$
$$A = \{(x,y) \in \mathbb{R}^2, y \geq 0, x \in [1,4], |xy| \leq 1\}$$

[R: $1/e, 0$]

(7)

$$f(x,y,z) = x$$
$$A = \{(x,y,z) \in \mathbb{R}^3 | y + z = 1, x^2 + y^2 + z^2 = \frac{5}{2}\}$$

[R:$\sqrt{2}, -\sqrt{2}$]

(8)

$$f(x,y) = |y - x^2|$$
$$A = \{(x,y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1, y \leq 1/2\}$$

[R: $5/4, 1/4$]