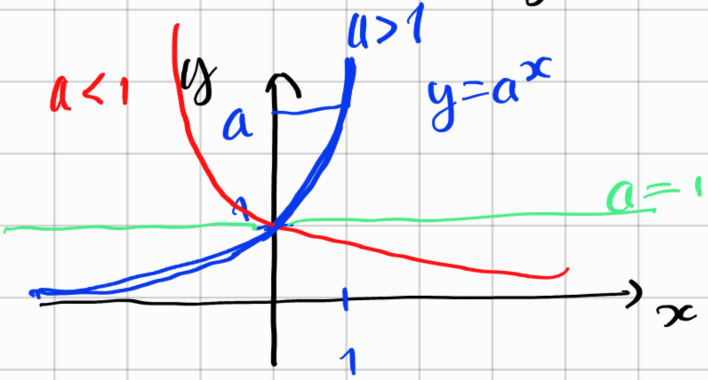
ANALISI MATEMATICA B
LEZIONE 14 – 24.10.2022

l'esponenziale

Se $a>1$ $\quad a^x$ $\quad \begin{cases} a^1 = a \\ a^{x+y} = a^x \cdot a^y \\ x \geq 0 \Rightarrow a^x \geq 1 \end{cases}$

Se $a=1$ $\quad a^1 = a$

Se $0<a<1$ $\quad \begin{cases} \\ x \geq 0 \Rightarrow a^x \leq 1 \end{cases}$



$f(x) = a^x$

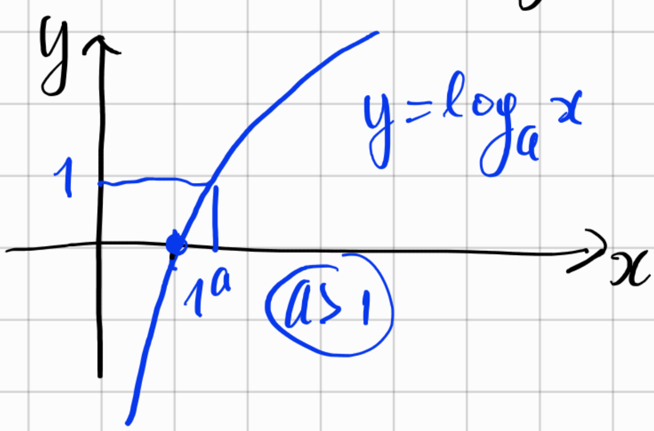
$f : \mathbb{R} \to \mathbb{R}_+ = \{x>0\}$

f è invertibile, f^{-1} è il logaritmo

$f^{-1}(x) = \log_a x$ $\quad \log_a(a^x) = x$

$f^{-1} : \mathbb{R}_+ \to \mathbb{R}$

$$\begin{cases} \log_a a = 1 \\ \log_a(x \cdot y) = \log_a x + \log_a y \\ x \geq 1 \Rightarrow \log_a x \begin{cases} \geq 0 & \text{se } a \geq 1 \\ \leq 0 & \text{se } a \leq 1 \end{cases} \end{cases}$$



$a \neq 1$

$$\log_a(b^x) = x \log_a b \qquad \log_a 1 = 0$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

POTENZA

prodotto ripetuto

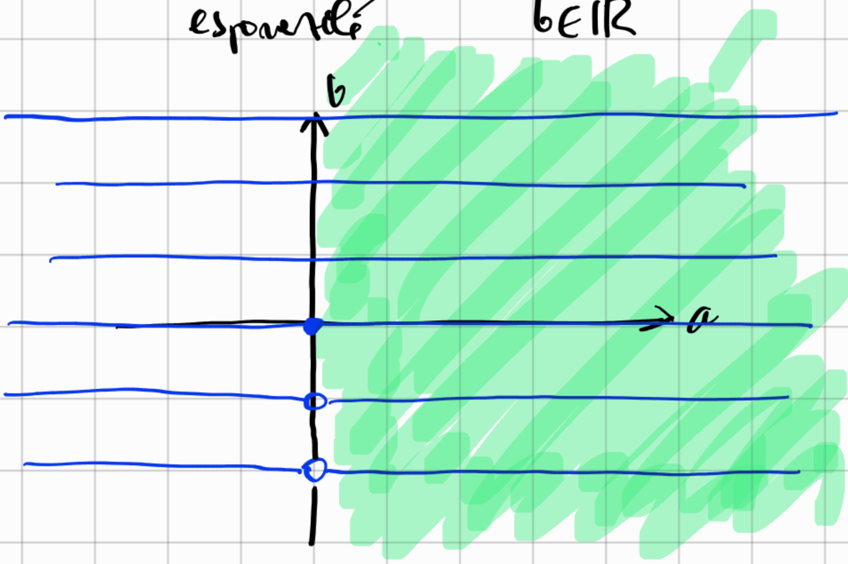
a^b

$b \in \mathbb{N} \quad a^b = \overbrace{a \cdot a \cdots a}^{b\text{-volte}} \qquad a \in \mathbb{R}$

$b \in \mathbb{Z} \quad a^{-1} = \frac{1}{a} \qquad a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n \qquad a \neq 0$

$a > 0$
$b \in \mathbb{R}$

esponenziali



SI POTREBBE DEFINIRE

$a^{1/3} = \sqrt[3]{a}$

$a^{2/3} = \sqrt[3]{a^2}$

$a^{3/2} = \sqrt[2]{a^3} \qquad a \geq 0$

Es $(-1)^{-3} = -1 \qquad \cancel{(-3)^{\sqrt{2}}}$

$(\sqrt{2})^{-3} = \frac{1}{2\sqrt{2}}$

Proprietà delle funzioni

GRAFICO

$f : A \to B$ $\qquad G_f = \{(x,y) : x \in A,\ y = f(x)\}$

$$G_f \subseteq A \times B$$

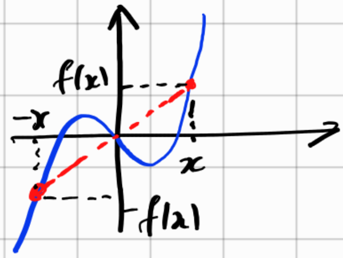
SIMMETRIE

f è **pari** se

$$f(x) = f(-x)$$

$\forall x \in A$. $\quad (A = -A, \quad x \in A \Rightarrow -x \in A)$

f è **dispari** se

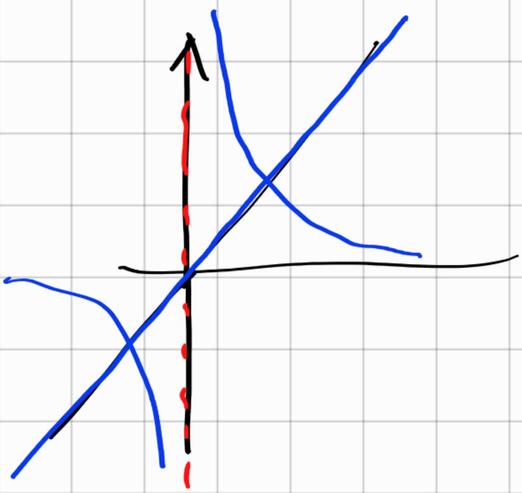


$$f(-x) = -f(x)$$

$(A = -A)$.

<u>ES</u> $\quad f(x) = x^n$ è pari se n pari
dispari se n dispari

$$(-x)^n = \begin{cases} x^n & \text{se } n \text{ pari} \\ -x^n & \text{se } n \text{ dispari} \end{cases}$$



<u>ES</u> $\quad f = f^{-1}$ $\qquad y = x$, $\quad y = \frac{1}{x}$

il grafico è simmetrico rispetto a $y = x$

MONOTONIA

$\underbrace{f: A \subseteq \mathbb{R} \to \mathbb{R}}$

def f si dice essere: $\forall x, y \in A$

[weakly increasing]

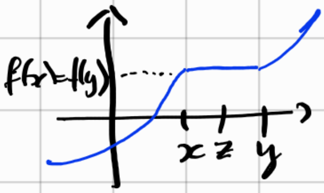
crescente: $x \leq y \Rightarrow f(x) \leq f(y)$

strettamente crescente: $x < y \Rightarrow f(x) < f(y)$

[increasing]

$\Downarrow$

iniettiva



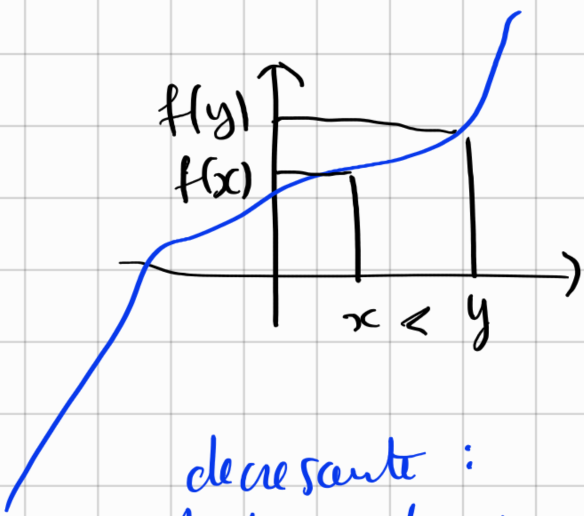
Oss f è crescente ma non strettamente crescente

$\Updownarrow$

f è crescente ed esistono $x<y$ tali che $\forall z: x<z<y$ si ha $f(z)=f(x)=f(y)$.

ES $f(x)=x$ è strettamente crescente

$f(x)=x^3$ "



$f(x)=a^x$ $(a>1)$

$f(x)=x^2$ non è crescente

$-1<0$

$(-1)^2 > 0^2$

decrescente: $x \leq y \Rightarrow f(x) \geq f(y)$

strettamente decrescente: $x<y \Rightarrow f(x) > f(y)$.

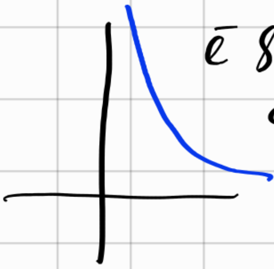
ES $f(x)$ crescente $\Leftrightarrow -f(x)$ decrescente

strett. crescente — strett. decrescente

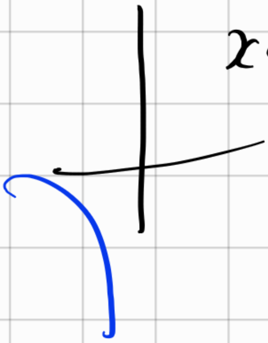
Es $f(x) = \frac{1}{x}$

non è decrescente
né crescente

Se $x>0$

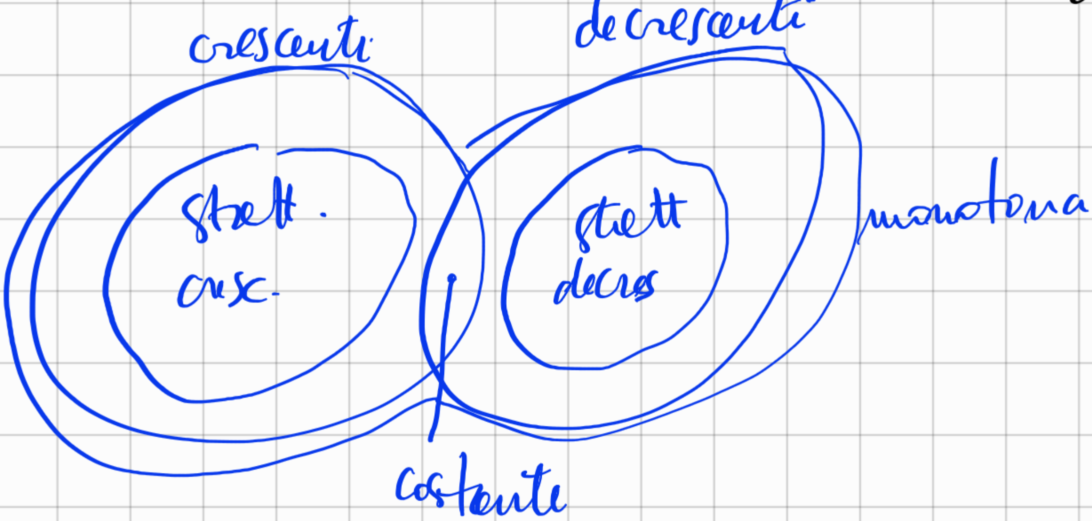


è strett.
decr.



monotona: se è crescente o decrescente

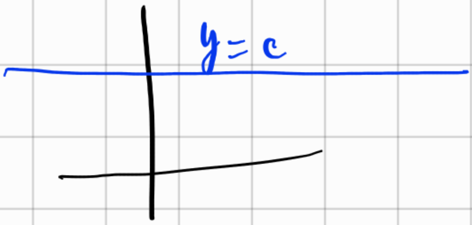
strett. monotona: se è strett crescente o strett. decrescente.



costante: se è crescente e decrescente

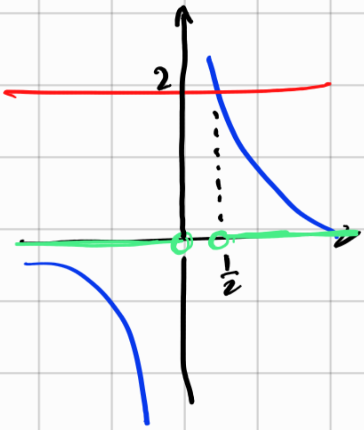
$$\left(x \le y \Rightarrow f(x) = f(y) \right)$$

$$\exists c : f(x) = c \quad \forall x.$$



$$\frac{1}{f(x)} < 2 \overset{f(x)>0}{\iff} f(x) > \frac{1}{2}$$

ES $\quad \frac{1}{x} < 2 \iff \begin{cases} x>0: & x > \frac{1}{2} \\ x<0: & \text{sempre verificata} \end{cases}$



$\iff x > \frac{1}{2}$ o $x < 0$.

$\Rightarrow$ per definizione

Oss $\quad f$ strett. crescente: $\quad x < y \iff f(x) < f(y)$

$\left[\; f(x) < f(y) \text{ e } x \geq y \Rightarrow x > y \Rightarrow f(y) < f(x) \;\right]$

f strett. decresc.: $\quad x < y \iff f(x) > f(y)$.

f strett. monotona $\Rightarrow f$ iniettiva $\quad x \neq y \underset{\Leftarrow}{\Rightarrow} f(x) \neq f(y)$

$$f(x) = \frac{1}{2^{\boxed{(x+1)^3+1}} + 7}$$

$f(x) > 2$

$f(x) = 3$

$f(x) \leq 100$

f è strett. decrescente.

$x \overset{f_1}{\longmapsto} x+1 \qquad x \overset{f_2}{\longmapsto} x^3 \qquad x \overset{f_3}{\longmapsto} 2^x \qquad x \overset{f_4}{\longmapsto} x+7$

$x \overset{f_5}{\longmapsto} \frac{1}{x}$

$$f(x) = f_5\left(f_4\left(f_3\left(f_1\left(f_2\left(f_1(x)\right)\right)\right)\right)\right)$$

strett. decr. — strett. cresc

Es

f	g	$f \circ g$
cresc	cresc	cresc.
cresc	decr	decr.
decr	cres	decr
decr	decr	cresc.

$$f^{-1}(y) = f_1^{-1}\left(f_2^{-1}\left(f_1^{-1}\left(f_3^{-1}\left(f_4^{-1}\left(f_5^{-1}(y)\right)\right)\right)\right)\right)$$

$$y = \frac{1}{2^{\boxed{[(x+1)^3+1]}} + 7}$$

$$\frac{1}{y} = 2^{[(x+1)^3+1]} + 7$$

$$\frac{1}{y} - 7 = 2^{[(x+1)^3+1]}$$

$$\log_2\left(\frac{1}{y} - 7\right) = (x+1)^3 + 1$$

$$\log_2\left(\frac{1}{y}-7\right)-1=(x+1)^3$$

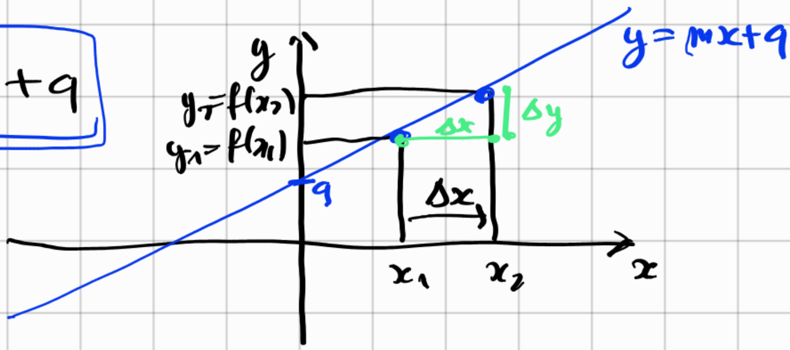
$$\sqrt[3]{\log_2\left(\frac{1}{y}-7\right)-1}=x+1$$

$$\sqrt[3]{\log_2\left(\frac{1}{y}-7\right)-1}-1=x$$

$$x=f_1^{-1}\left(f_2^{-1}\left(f_1^{-1}\left(f_3^{-1}\left(f_4^{-1}\left(f_5^{-1}(y)\right)\right)\right)\right)\right)$$

FUNZIONI LINEARI (AFFINI)

$$\boxed{f(x)=\underbrace{m\cdot x+q}}$$



$$f(x_2)-f(x_1)=(m x_2+q)-(m x_1+q)=m(x_2-x_1)$$

$$m=\frac{f(x_2)-f(x_1)}{x_2-x_1} \leftarrow \text{rapporto incrementale.}$$

$$m=\frac{\Delta y}{\Delta x}$$

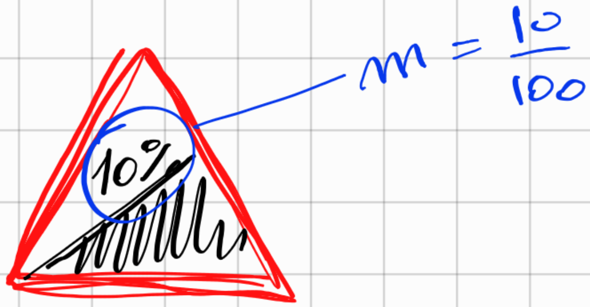
$\Delta x = x_2-x_1$

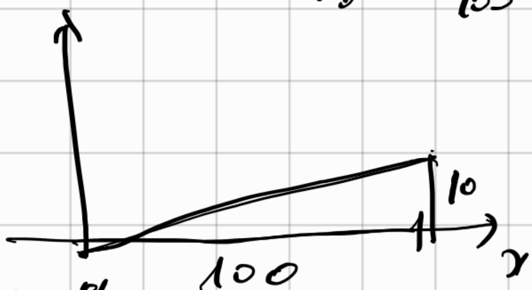
$\Delta y = y_2-y_1 = f(x_2)-f(x_1)$

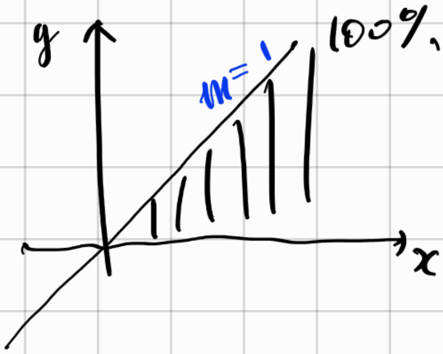
$\Delta f(x)$

$m=$ "pendenza"

$\% = \frac{1}{100}$







$m>0$ f è strett crescente

$m=0$ $f(x)=q$ costante

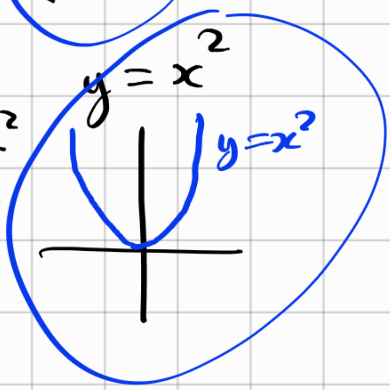
$m<0$ f è strett. decrescente

$y=mx+q$

$x = \frac{x-q}{m} = \frac{1}{m}x - \frac{q}{q}$

FUNZIONI QUADRATICHE

$$y=f(x) = \underbrace{ax^2+bx+c} \qquad (a\neq 0)$$



$$0 = ax^2+\underbrace{bx}+c \qquad (x+d)^2 = x^2+2dx+d^2$$

$$= a\left(x^2+\underbrace{\frac{b}{a}x}+\frac{c}{a}\right)$$

$$= a\left(x^2+2\cdot\frac{b}{2a}x+\left(\frac{b}{2a}\right)^2+\frac{c}{a}-\left(\frac{b}{2a}\right)^2\right)$$

$$= a\left(\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right)$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2} \qquad (2ax+b)^2 = b^2-4ac=\Delta$$

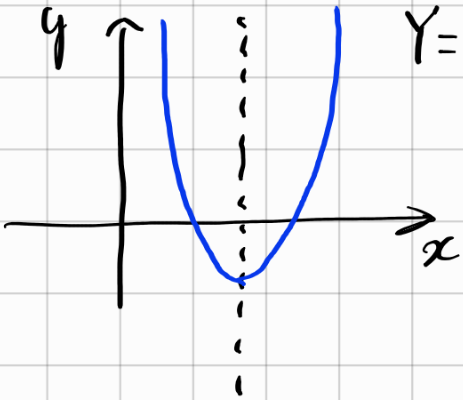
$$X=2ax+b \qquad X^2=\Delta$$

Se $\Delta > 0$ ho 2 sol. $X = \pm\sqrt{Y}$

$2ax + b = \pm\sqrt{\Delta}$ $\quad x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$

Se $\Delta = 0$ ho 1 sol. $X = 0$ $\quad x_{1,2} = \frac{-b}{2a}$

Se $\Delta < 0$ non ci sono sol.



$Y = X^2 - \Delta$

OSSERVAZIONE

C'è una unica parabola a meno di isometrie e riscalamenti.